Applying Thevenin's theorem at the terminals X, Y in fig (2) and looking ~~then~~ towards left, the current source $h_{fe} I_1$ in parallel with the resistance $(1/h_{oe})$ can be replaced by an equivalent Thevenin voltage source of generated voltage $(h_{fe} I_1 / h_{oe})$ and internal resistance $1/h_{oe}$. This Thevenin representation is shown in fig (3).

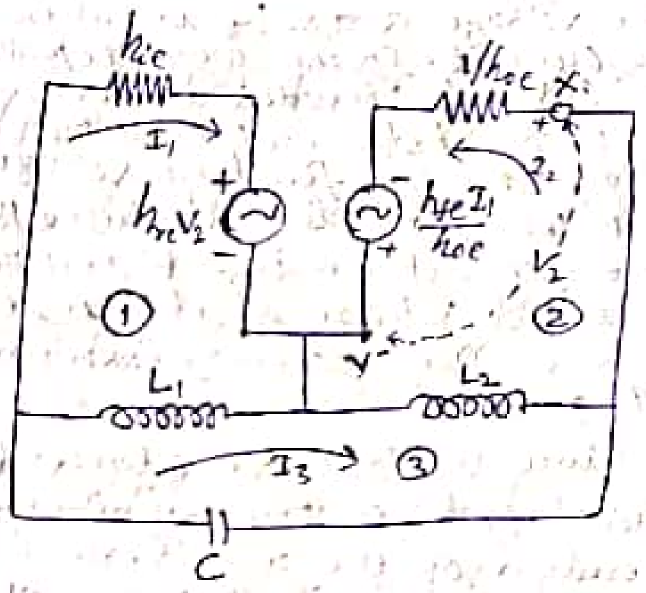

Fig (3)

For simplicity, the mutual inductance between L_1 and L_2 is neglected.

From fig (3), we find that, the voltage across XY, i.e, $V_2 = \frac{1}{h_{oe}} I_2 - \frac{h_{fe}}{h_{oe}} I_1 \longrightarrow (1)$

Applying Kirchhoff's voltage law equations for loop (1), (2) and (3), we have three equations.

For loop-1, we obtain

$$(h_{ie} + jX_{L_1}) I_1 + h_{re} V_2 - jX_{L_1} I_3 = 0$$

or, $(h_{ie} + jX_{L_1}) I_1 + h_{re}\left(\frac{1}{h_{oe}} I_2 - \frac{h_{fe}}{h_{oe}} I_1\right) - jX_{L_1} I_3 = 0$ [from (1)]

or, $\left(h_{ie} + jX_{L_1} - \frac{h_{fe}h_{re}}{h_{oe}}\right) I_1 + \frac{h_{re}}{h_{oe}} I_2 - jX_{L_1} I_3 = 0 \longrightarrow (2)$

For loop (2),

$$-\frac{h_{fe}}{h_{oe}} I_1 + \left(\frac{1}{h_{oe}} + jX_{L_2}\right) I_2 + jX_{L_2} I_3 = 0 \longrightarrow (3)$$

And for loop-(3);

$$-jX_{L_1} I_1 + jX_{L_2} I_2 + (jX_{L_1} + jX_{L_2} - jX_C) I_3 = 0 \longrightarrow (4)$$

where $X_{L_1} = \omega L_1$, $X_{L_2} = \omega L_2$ and $X_C = \frac{1}{\omega C}$;

ω = angular frequency of oscillation.

The currents I_1, I_2 and I_3 are non-zero (Non-trivial sol^n). Therefore the non-trivial solution of equations (2), (3) and (4) exist if the determinant of co-efficients of I_1, I_2, I_3 is zero.

that is

$$\begin{vmatrix} \left(h_{ie}+jX_{L_1}-\frac{h_{fe}h_{re}}{h_{oe}}\right) & \left(\frac{h_{re}}{h_{oe}}\right) & (-jX_{L_1}) \\ \left(-\frac{h_{fe}}{h_{oe}}\right) & \left(\frac{1}{h_{oe}}+jX_{L_2}\right) & (jX_{L_2}) \\ (-jX_{L_1}) & (jX_{L_2}) & (jX_{L_1}+jX_{L_2}-jX_c) \end{vmatrix}=0 \longrightarrow (5)$$

At the frequency of oscillations it can be assumed that

$$jX_{L_1}+jX_{L_2}-jX_C=0 \quad \text{-----} (6)$$

$$\left(h_{ie}+jX_{L_1}-\frac{h_{fe}h_{re}}{h_{oe}}\right)\left[\left(\frac{1}{h_{oe}}+jX_{L_2}\right)(jX_{L_1}+jX_{L_2}-jX_c)+X_{L_2}^2\right]$$
$$+\frac{h_{re}}{h_{oe}}\left[\frac{h_{fe}}{h_{oe}}(jX_{L_1}+jX_{L_2}-jX_c)+X_{L_1}X_{L_2}\right]$$
$$-\frac{h_{fe}}{h_{oe}}X_{L_1}X_{L_2}+\left(\frac{1}{h_{oe}}+jX_{L_2}\right)X_{L_1}^2=0 \longrightarrow (7)$$

On solving equation (7), we have.

$$\left[\left(\frac{h_{ie}}{h_{oe}}-\frac{h_{fe}h_{re}}{h_{oe}^2}-X_{L_1}X_{L_2}\right)+j\left(\frac{1}{h_{oe}}X_{L_1}+h_{ie}X_{L_2}-\frac{h_{fe}h_{re}}{h_{oe}}X_{L_2}\right)\right]$$
$$\times(jX_{L_1}+jX_{L_2}-jX_c)+h_{ie}X_{L_2}^2+jX_{L_1}X_{L_2}^2-\frac{h_{fe}h_{re}}{h_{oe}}X_{L_2}^2$$
$$+\frac{h_{fe}h_{re}}{h_{oe}^2}(jX_{L_1}+jX_{L_2}-jX_c)+\frac{h_{re}}{h_{oe}}X_{L_1}X_{L_2}$$
$$-\frac{h_{fe}}{h_{oe}}X_{L_1}X_{L_2}+\frac{1}{h_{oe}}X_{L_1}^2+jX_{L_2}X_{L_1}^2=0 \longrightarrow (8)$$

Equating real part of eq^n (8) to zero and using equation (6), we have

$$h_{ie}X_{L_2}^2-\frac{h_{fe}h_{re}}{h_{oe}}X_{L_2}^2+\frac{h_{re}}{h_{oe}}X_{L_1}X_{L_2}-\frac{h_{fe}}{h_{oe}}X_{L_1}X_{L_2}+\frac{1}{h_{oe}}X_{L_1}^2=0$$

or

$$\frac{(h_{ie}h_{oe}-h_{fe}h_{re})}{h_{oe}}X_{L_2}^2+\left(\frac{h_{re}-h_{fe}}{h_{oe}}\right)X_{L_1}X_{L_2}+\frac{1}{h_{oe}}X_{L_1}^2=0$$

or

$$(h_{ie}h_{oe}-h_{fe}h_{re})X_{L_2}^2+(h_{re}-h_{fe})X_{L_1}X_{L_2}+X_{L_1}^2=0$$

or

$$\Delta_{he}X_{L_2}^2+(h_{re}-h_{fe})X_{L_1}X_{L_2}+X_{L_1}^2=0$$

where $\Delta_{he}=h_{ie}h_{oe}-h_{fe}h_{re}$. Since $h_{re}\ll h_{fe}$, we have

$$\Delta_{he}X_{L_2}^2-h_{fe}X_{L_1}X_{L_2}+X_{L_1}^2=0$$

This is quadratic in X_{L_2}. Therefore,

$$X_{L_2}=\left[\frac{h_{fe}\pm\sqrt{h_{fe}^2-4\Delta_{he}}}{2\Delta_{he}}\right]X_{L_1}$$

Again, $4\Delta_{he}\ll h_{fe}^2$, therefore,

$$X_{L_2}=\frac{h_{fe}}{\Delta_{he}}X_{L_1}$$

or

$$\omega L_2=\frac{h_{fe}}{\Delta_{he}}\omega L_1$$

or $\quad L_2 = \frac{h_{fe}}{\Delta h_e} L_1 \longrightarrow (9)$

This is the required condition for sustained oscillation.

FREQUENCY OF OSCILLATOR

Equating imaginary part of eqn (8) to zero we have,

$$\frac{h_{ie}}{h_{oe}}(X_{L_1}+X_{L_2}-X_C) - \frac{h_{fe}h_{re}}{h_{oe}}(X_{L_1}+X_{L_2}-X_C) + X_{L_1}X_{L_2}^2$$
$$- X_{L_1}X_{L_2}(X_{L_1}+X_{L_2}-X_C) + X_{L_2}X_{L_1}^2 = 0$$
$$+ \frac{h_{fe}h_{re}}{h_{oe}^2}(X_{L_1}+X_{L_2}-X_C)$$

or, $\frac{h_{ie}}{h_{oe}}(X_{L_1}+X_{L_2}-X_C) + X_{L_1}X_{L_2}X_C = 0$

or, $(X_{L_1}+X_{L_2}-X_C) + \frac{h_{oe}}{h_{ie}} X_{L_1}X_{L_2}X_C = 0$

or, $\left(\omega L_1+\omega L_2-\frac{1}{\omega C}\right) + \frac{h_{oe}}{h_{ie}}\cdot\omega^2 L_1L_2\cdot\frac{1}{\omega C} = 0$

or, $\left(L_1+L_2-\frac{1}{\omega^2 C}\right) + \frac{h_{oe}}{h_{ie}}\frac{L_1L_2}{C} = 0$

or, $\frac{1}{\omega^2 C} = (L_1+L_2) + \frac{h_{oe}}{h_{ie}}\frac{L_1L_2}{C}$

or, $\frac{1}{\omega^2} = (L_1+L_2)C + \frac{h_{oe}}{h_{ie}}L_1L_2$

$$\omega^2 = \frac{1}{\frac{h_{oe}}{h_{ie}}L_1L_2 + (L_1+L_2)C}$$

$$\therefore \omega = \frac{1}{\sqrt{\frac{h_{oe}}{h_{ie}}L_1L_2+(L_1+L_2)C}}$$

or, $f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\frac{1}{\sqrt{\frac{h_{oe}}{h_{ie}}L_1L_2+(L_1+L_2)C}} \longrightarrow (10)$

In actual circuit, $\frac{h_{oe}}{h_{ie}}L_1L_2 \ll (L_1+L_2)C$.

$\therefore f = \frac{1}{2\pi\sqrt{(L_1+L_2)C}} = \frac{1}{2\pi\sqrt{LC}} \longrightarrow (11)$

where $L = L_1+L_2$ is the total induction of the tank circuit. In deriving eqn (11), we have neglected the mutual inductance M of the coil L_1 and L_2. If M is also taken into consideration the expression for frequency is obtained by replacing L_1 by L_1+M and L_2 by L_2+M. If this is done the frequency becomes

$$f = \frac{1}{2\pi\sqrt{(L_1+L_2+2M)C}} \longrightarrow (12)$$

Thus the frequency of the Hartley oscillator can be varied by changing L or C. This oscillator is a radio frequency generator and is usually used in radio receivers.