(1)

UG-I

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PROPERTIES OF PROGRESSIVE WAVE

The equation of the plane-progressive wave is given by

$$y = a \sin \frac{2\pi}{\lambda}(vt - x) \quad \text{---(1)}$$

Where v = velocity of wave = $\frac{\lambda}{T}$ and x = the ~~displace~~ distance from the fixed point. y = Displacement of the particle and a is the amplitude of wave.

The above equation shows the following properties of wave motion.

(i) When x is constant, i.e at fixed point, y is a periodic function of t. It means that all particles of the medium vibrate simple harmonically with the same amplitude and period.

(ii) For the different positions i.e as x increases the phase angle decreases. It means the phase changes from particle to particle.

(iii) When t is constant, y is a function of x. It means that at particular instant the displacement varies periodically from particle to particle.

If x is increased by λ, then the displacement is given by

$$y' = a \sin \frac{2\pi}{\lambda}\{vt - (x+\lambda)\}$$

$$= a \sin\left\{\frac{2\pi}{\lambda}(vt - x) - 2\pi\right\}$$

$$= a \sin \frac{2\pi}{\lambda}(vt - x) \qquad \because \sin(\alpha - 2\pi) = \sin\alpha$$

Thus at any instant the displacement of

particles at a distance λ apart is the same. Hence such particles are in the same phase of vibration.

(iv) When both x & t change, let t increase by δt and x by $v\delta t$. Then the displacement is given by

$$y' = a \sin \frac{2\pi}{\lambda}\{v(t+\delta t) - (x+v.\delta t)\}$$
$$= a \sin \frac{2\pi}{\lambda}\{vt + v\delta t - x - v\delta t\}$$
$$= a \sin \frac{2\pi}{\lambda}(vt - x) = y$$

It means that the disturbance at a place x at an instant t is transferred to another place at a distance $v.\delta t$ away in time δt. That is ~~advancing ca~~ the disturbance is advancing with a velocity v without suffering any change in shape.

(v) From eq^n (1) we can very easily show that
$$\frac{dy}{dt} = -v\frac{dy}{dx} \quad \ldots\ldots (2)$$

For the particles moving forward ($\frac{dy}{dt} = +ve$), $\frac{dy}{dx}$ is negative i.e the particles are compressed; but for the particle moving backward ($\frac{dy}{dt}$ negative), $\frac{dy}{dx}$ is positive i.e. the particles are "rarefied".

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