OPERATORS :-

Any operator is a symbol for a rule for transforming a given mathematical function into another function. It has no any physical meaning if written alone.

for example -

d/dx is an operator which transforms a function into its first derivative w.r.t 'x' as -

$\frac{d}{dx}$ transform the function $\sin x$ into the function $\cos x$.

In general, if $\hat{A}$ denotes an operator which transform the function $f(x)$ in to the ~~f(x)~~ function $g(x)$.

i.e. $\hat{A} f(x) = g(x)$.

(1) Let $\hat{A} = \frac{d}{dx}$

& $f(x) = ax^2$

then, $\hat{A} f(x) = \frac{d}{dx}(ax^2) = 2ax$.

i.e. $g(x) = 2ax$.

(2) Let $\hat{A} = a$

& $f(x) = x^2 + c$

then, $\hat{A} f(x) = a(x^2+c) = ax^2 + ac$

i.e. $g(x) = ax^2 + ac$.

An operator can be added, substracted, multiplied and have some other properties.

An operator is thus, an instruction that carrying out certain operation. An operator is written with a cap sign (^) over head.

e.g.

Hamiltonian operator = $\hat{H}$.

Angular momentum operator :-

The Angular momentum operator ($\vec{L}$) is expressed in the terms of position vector ($\vec{r}$) and linear momentum vector ($\vec{p}$) as.

$$\vec{L} = \vec{r} \times \vec{p}$$

if $\vec{i}$, $\vec{j}$ and $\vec{k}$ are unit vectors along x, y & z co-ordinates respectively then –

$$\vec{r} = \vec{i}x + \vec{j}y + \vec{k}z$$

and $$\vec{p} = \vec{i}p_x + \vec{j}p_y + \vec{k}p_z$$

Thus, $$\vec{L} = (\vec{i}x + \vec{j}y + \vec{k}z) \times (\vec{i}p_x + \vec{j}p_y + \vec{k}p_z)$$
$$= \vec{i}(yp_z - zp_y) + \vec{j}(zp_x - xp_z) + \vec{k}(xp_y - yp_x)$$

or $$\vec{L} = \vec{i}L_x + \vec{j}L_y + \vec{k}L_z$$

Now taking the coefficient of i, j & k from both parts separately. Thus,

$$L_x = (y p_z - z p_y) = \frac{h}{2\pi i}\left(y\frac{d}{dz} - z\frac{d}{dy}\right)$$

$$L_y = (z p_x - x p_z) = \frac{h}{2\pi i}\left(z\frac{d}{dx} - x\frac{d}{dz}\right)$$

$$\& \quad L_z = (x p_y - y p_x) = \frac{h}{2\pi i}\left(x\frac{d}{dy} - y\frac{d}{dx}\right)$$

Where, 'i' is an imaginary quantity.

i.e. $i = \sqrt{\pm 1}$.

* Linear and non-linear operator.

Linear operator.

An operator while operating the sum of two function and gives the same results as the sum of operations of the two ~~xxx~~ functions seperately then, it is called linear operator.

for example-

The differential $\frac{d}{dx}$ is a linear operator

i.e. $$\frac{d}{dx}\left[f(x) + g(x)\right] = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$$

Where, $f(x)$ & $g(x)$ are two functions.

$\frac{d}{dx}$ is operator called linear operator.

Non-linear operator :-

An operator while operating the sum of two functions gives different results from the sum of the operation of two function seperately. then it is called non. linear operator.

for example—

If $f(x)$ and $g(x)$ are two functions then, its square root operator is non-linear operator.

i.e. $\sqrt{f(x)+g(x)} \neq \sqrt{f(x)} + \sqrt{g(x)}$

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