Write Notes on Any one of the three

(1) CONSTRAINTS.

The limitations or geometrical restrictions on the motion of a particle or system of particles are generally known as Constraints.

A Particle needs three independent Parameters (degrees of freedom) to specify its Position in space. If the Particle is assumed to be moving on table top, its motion is Confined to the Surface of the table top only and it requires two Parameters to locate its Position on the top. In this case its own motion is said to be restricted or limited. Then, it is said to be under Constraint. Thus the Constraints reduce the number of independent Coordinates (degrees of freedom). ~~the~~ Similarly, in the motion of a Particle moving on a st. line, there are two Constraint Present.

Classification of Constraints.

If the Conditions of Constraint are expressed as equations Connecting the Coordinates of Particle having the form

$$f(\vec{r}_1, \vec{r}_2 \cdots\cdots, t) = 0$$

then the Constraint are said to be holonomic. ~~As~~ for example, the Constraints of a rigid body may be given by

$$(\vec{r}_i - \vec{r}_j)^2 - C_{ij}^2 = 0$$

For another example, the motion of Point mass of a Simple Pendulum is restricted, since the distance between the Point mass and Point of Suspension is Constant. The Condition of Constraint are expressed as

$(\vec{r} - \vec{a})^2 = l^2$ where $\vec{r}$ is the Position vector of the Point mass & $\vec{a}$ is the Position vector of Point of Suspension.

The Constraints which can't be expressed in this fashion are Called <u>Non-holonomic</u>. As for example, the motion of the gas molecules within the Container is restricted by the walls of the vessel. Another example, the motion of a Particle Placed on the Surface of sphere, which may be expressed as $r^2 - a^2 \geq 0$, where a is the radius of the sphere.

If the Constraints are independent of time, they are termed as Schleronomic, but if they Contain time explicitly, they are Called Rheonomic. A bead Sliding on a moving wire is an example of Rheonomic Constraints.

(2) Generalised Co-ordinates

Any set of independent Co-ordinates (or variables) sufficient in number to define unambiguously the System Configuration is Called generalised Coordinates.

These generalised Coordinates may Consist of Cartesian Coordinates, Polar Coordinates, Spherical Polar Coordinates. These are generally denoted by $q_1, q_2, q_3 \ldots q_f$, f refers to the number of degrees of freedom.

To define the Position of a System of N-Particles in space, 3N independent Coordinates or degrees of freedom are required. If Constraints are expressed by P equations, then these equations may be used to eliminate P of the 3N Coordinates thereby leading 3N−P independent Coordinates with 3N−P degrees of freedom of the System. Thus we have 3N−P independent variables $q_1, q_2 \ldots q_{3N-P}$ into terms of which the old coordinates $r_1, r_2 \ldots r_n$ Can be expressed as

$$\vec{r}_1 = \vec{r}_1 (q_1, q_2 \ldots q_{3N-P}, t)$$
$$\vec{r}_2 = \vec{r}_2 (q_1, q_2 \ldots q_{3N-P}, t)$$
$$\vec{r}_N = \vec{r}_N (q_1, q_2 \ldots q_{3N-P}, t)$$

Here $q_1, q_2 \ldots q_{3N-p}$ are called generalised coordinates of the Particle. The number $K = 3N-p$ is called the no. of degrees of freedom of the System.

The Configuration ~~space~~ of the System may be represented by the Position of a Point in a K-dimensional space which is called the Configuration space of the system.

The generalised Coordinates of the functions of N-variables $\vec{r}_1, \vec{r}_2 \ldots \vec{r}_N$ and one time variable t, are given by

$$q_i = q_i(\vec{r}_1, \vec{r}_2, \ldots r_N, t)$$

where $i = 1, 2 \ldots k$,

They are independent of each other. By use of the generalised coordinates the general 3N dimensional Configuration ~~spa~~ is reduced to 3N−P dimensional space & get rid of forces due to Constraints.

Elimination of Constraints in this manner reduces the number of Coordinates to a minimum.

(3) D' Alembert's Principle

Let any no. of forces be applied to the i th Particle of a system at any instant of time. The virtual displacement of a system refers to a change in the Configuration of the system. If the system is in equilibrium, then the total force on each Particle is zero i.e. $F_i = 0$

So $\vec{F}_i \cdot \delta\vec{r}_i = 0$,

where $\delta\vec{r}_i$ is virtual displacement and F_i is the force in equilibrium on the ith Particle. Since sum of these vanishing product over all Particles must likewise be zero, therefore

$$\sum \vec{F_i} \cdot \delta \vec{r_i} = 0.$$

This means that if system of free Particles is equilibrium, the total work done by all forces, internal and external, in a virtual displacement is zero.

The force $\vec{F_i}$ is written as

$\vec{F_i} = \vec{F_i}^a + \vec{f_i}$, f_i being force of Constraint, therefore the above equation becomes

$$\sum_i \vec{F_i}^a \cdot \delta \vec{r_i} + \sum_i \vec{f_i} \cdot \delta \vec{r_i} = 0$$

If we take force of Constraints normal to motion then $\vec{f_i} \cdot \delta \vec{r_i} = 0$ since $\delta \vec{r_i} \perp r \vec{f_i}$.

Then we have $\sum \vec{F_i}^a \cdot \delta \vec{r_i} = 0$.

Thus for equilibrium of a system virtual work of the applied force vanishes. This is known as Principle of virtual work.

This Principle was modified into a new Principle by J. Bernoulli & developed by D' Alembert by taking

$\vec{F_i} = \dot{\vec{P_i}}$ then $\vec{F_i} - \dot{\vec{P_i}} = 0$

which states that the Particle in the system will be in equilibrium under a force equal to the actual force plus a reversed effective force $-\dot{P_i}$. Thus the above equation takes the form

$$\sum_i (\vec{F_i} - \dot{\vec{P_i}}) \cdot \delta \vec{r_i} = 0$$

or $$\sum_i (\vec{F_i}^a - \dot{P_i}) \cdot \delta \vec{r_i} + \sum_i \vec{f_i} \cdot \delta \vec{r_i} = 0$$

If the virtual work of forces of Constraint vanishes, then

$$\sum_i (\vec{F}_i^a - \dot{\vec{P}}_i) \cdot \delta \vec{r}_i = 0$$

which is known as D'Alembert's Principle where the forces due to Constraint disappear and we may use $\vec{F}_i$ in place of $\vec{F}_i^a$ without ambiguity.