Frame of Reference

Experiments as well as simple observations reveal that the motion is only relative. Absolute motion has no physical meaning. At best the motion of one body is described relative to any other well defined system or body. This well defined system relative to which the motion of an object described is called a "frame of reference." For instance, the motion of a flying aircraft is specified w.r.to the coordinate system fixed on the earth; the motion of a charged particle in a particle accelerator is given relative to the accelerator. Here earth and accelerator are the frames of reference.

Broadly, there are two kinds of frame of references:

ⓐ Inertial frame of reference
& ⓑ Non-inertial frame of reference.

Inertial frame of reference

The frame relative to which the body is either at rest or moving with uniform, linear velocity ~~are~~ is known as inertial frame of reference.

Or Two frames can be said to be ~~of~~ inertial frames of reference with respect to one another when they are either at rest or in uniform relative motion with respect to one another.

In such frames, space is homogeneous and isotropic and time is also homogeneous. In particular, in such a frame a free body at rest at any instant will remain always at rest or a body moves with constant velocity in absence of any external force.

Thus, we see that an inertial frame is one in which Law of inertia or Newton's first Law of motion is valid.

Another Property ~~for de~~ that can be utilized for defining inertial frames is the one according to which the equation of motion of a body takes on the simplest form.

The equations of motion of a Particle are invariant under Galilean transformation since they preserve their form when transformed from one inertial frame to another inertial frame moving with uniform velocity.

Non-Inertial frame of reference:

The frames relative to which the body not acted upon by external force, is accelerated are called non-inertial frames.

It clearly shows that newton's Law of motion does not hold true. From experimental inferences, we know that any-thing capable of rotating must have acceleration even in absence of external force. On this basis we can say that the earth is not an inertial frame. Because earth is rotating. Hence any system fixed in earth is also non-inertial frame.

It can be easily shown that any frame moving with constant angular velocity relative to any inertial frame, is also non-inertial. Instead, if any frame is in translational accelerated motion relative to inertial frame, is accelerated or non-inertial frame.

Let us now discuss the two cases in detail

Reference frame with translational Acceleration:

Let us consider two non-inertial frames S and S' such that the frame S' is moving with acceleration $\vec{a}_0$ with respect to S. Let a Particle has an acceleration $\vec{a}$ with respect to S. Then to the observer in S', it will appear to have acceleration $\vec{a}'$ given by $\vec{a}' = \vec{a} - \vec{a}_0$

Hence the force, $\vec{F}' = m\vec{a}' = m(\vec{a} - \vec{a}_0)$

$$= \vec{F} - m\vec{a}_0 = \vec{F} - \vec{F}_0$$

where $\vec{F}$ is the force seen by an observer in S and $\vec{F}_0$ is the force due to relative acceleration $\vec{a}_0$ between two frames.

when $\vec{F} = 0$, we get $\vec{F}' = -\vec{F}_0$

Thus, the Particle seems to experience a force, $-\vec{F}_0$ when viewed from S' even when there is no force on it in S. Evidently an accelerated frame is a non-inertial frame & force $\vec{F}_0$ is called the fictitious or Pseudo force which arises from the acceleration of reference frame.

Uniformly Rotating Frame : Coriolis and Centrifugal force :

In this case a), two Conditions may arise -

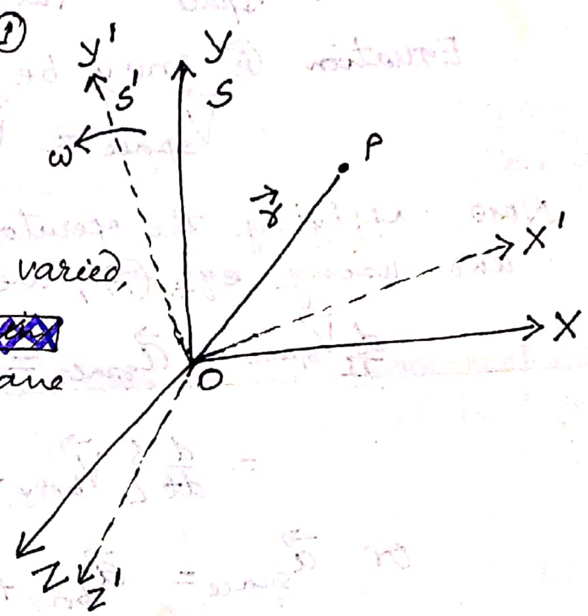
One is - when the origins of two frames Coincide and the

Other is - when the origins of two frames do not Coincide.

Let us Consider two frames S and S′ in which S′ is in uniform rotation w.r. to S at rest. Let the origins of the two frames Coincide at O. Also, let $\vec{r}$ be the radius vector of a Particle P moving in rotating frame.

then $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ $\quad$ ---→ ①

$= x'\vec{i}' + y'\vec{j}' + z'\vec{k}'$



where $\vec{i}, \vec{j}, \vec{k}$ are unit vectors in S frame and $\vec{i}', \vec{j}'$ & $\vec{k}'$ are the unit vectors in S′ frame.

In rotation unit vectors are varied,

So [illegible] we have

$$\frac{d\vec{r}}{dt} = \frac{d}{dt}(x'\vec{i}' + y'\vec{j}' + z'\vec{k}')$$

$$= \frac{dx'}{dt}\vec{i}' + \frac{dy'}{dt}\vec{j}' + \frac{dz'}{dt}\vec{k}'$$

$$+ x'\frac{d\vec{i}'}{dt} + y'\frac{d\vec{j}'}{dt} + z'\frac{d\vec{k}'}{dt} \quad \text{—— ②}$$

The first three terms in equation ② represent the velocity relative to S and remaining three terms gives the velocity of a Point attached to S′

If $\vec{\omega}$ is the angular velocity of S′ frame relative to S, then, according to $\vec{v} = \frac{d\vec{r}}{dt} = \vec{\omega} \times \vec{r}$, we get

$$\frac{d\vec{i}'}{dt} = \vec{\omega} \times \vec{i}', \quad \frac{d\vec{j}'}{dt} = \vec{\omega} \times \vec{j}' \text{ and } \frac{d\vec{k}'}{dt} = \vec{\omega} \times \vec{k}' \quad \text{—— ③}$$

Now Putting the values of eqn ③ in eqn (2), we obtain

$$\frac{d\vec{r}}{dt} = \frac{dx'}{dt}\vec{i}' + \frac{dy'}{dt}\vec{j}' + \frac{dz'}{dt}\vec{k}' + x'(\vec{\omega} \times \vec{i}') + y'(\vec{\omega} \times \vec{j}') + z'(\vec{\omega} \times \vec{k}') \quad \text{—— ④}$$

Equation ④ may be written as follows

$$\left(\frac{d\vec{r}}{dt}\right)_{space} = \left(\frac{d\vec{r}}{dt}\right)_{body} + \vec{\omega}\times\vec{r} \quad \text{———} \quad (5)$$

where $\left(\frac{d\vec{r}}{dt}\right)_{space}$ is the velocity of the rigid body with respect to S and $\left(\frac{d\vec{r}}{dt}\right)_{body}$ is the velocity with respect to S'. This result is actually true for any vector and can be represented by the following operator equation

$$\left(\frac{d}{dt}\right)_{space} = \left(\frac{d}{dt}\right)_{body} + (\vec{\omega}\times\ ) \quad \text{———} \quad (6)$$

Equation ⑥ may be written as

$$\vec{V}_{space} = \vec{V}_{body} + \vec{\omega}\times\vec{r} \quad \text{—} \quad (7)$$

Now, applying the operator equation ⑥ to the vector $\vec{V}_{space}$ and using eqn ⑦, we get

$$\frac{d}{dt}\vec{V}_{space} = \vec{a}_{space} = \left(\frac{d\vec{V}_{space}}{dt}\right)_{body} + \vec{\omega}\times V_{space}$$

$$= \frac{d}{dt}\left\{\vec{V}_{body} + (\vec{\omega}\times\vec{r})_{body}\right\} + \vec{\omega}\times(\vec{V}_{body} + \vec{\omega}\times\vec{r})$$

or $$\vec{a}_{space} = \vec{a}_{body} + 2(\vec{\omega}\times V_{body}) + \vec{\omega}\times(\vec{\omega}\times\vec{r}) + \frac{d\vec{\omega}}{dt}\times\vec{r} \quad \longrightarrow (8)$$

The acceleration in the rotating frame S' is

$$\vec{a}_{body} = \vec{a}_{space} - 2(\vec{\omega}\times\vec{V}_{body}) - \vec{\omega}\times(\vec{\omega}\times\vec{r}) - \frac{d\vec{\omega}}{dt}\times\vec{r} \quad \text{———} \quad (9)$$

The equation of motion in the fixed space is

$$\vec{F}_{space} = m\vec{a}_{space}$$

Hence $$m\vec{a}_{body} = F_{space} - 2m(\vec{\omega}\times\vec{V}_{body}) - m\vec{\omega}\times(\vec{\omega}\times\vec{r}) - m\frac{d\vec{\omega}}{dt}\times\vec{r} \quad \text{———} \quad (10)$$

Obviously to an observer in the rotating frame, the body appears to be moving under the effective force $m\vec{a}_{body}$. The term $m\vec{\omega}\times(\vec{\omega}\times\vec{r})$ in eqn ⑩ is the ordinary centrifugal force and is perpendicular to $\vec{\omega}$. Its magnitude is $m\omega^2 r\sin\theta$

The term $2m(\vec{\omega} \times \vec{V}_{body})$ is called Coriolis force and is perpendicular to both $\vec{\omega}$ and $\vec{V}_{body}$. This is nonzero only when $\vec{V}_{body} \neq 0$ and the velocity of a point relative to the rotating frame must have a nonzero projection on a plane perpendicular to the axis of rotation. The last term $\left(\frac{d\vec{\omega}}{dt} \times \vec{r}\right)$ is nonzero only when $\frac{d\omega}{dt} \neq 0$ and will vanish when ω is constant.

It is now clear that the rotating frame is a non-inertial frame because the particle is acted upon by two fictitious forces as centrifugal and Coriolis force in addition to real force $F_{space} = F_{body}$. In case of inertial frame the motion has the simplest form $\vec{F} = m\vec{a}$.

Effect of Centrifugal force

The earth is rotating from west to east and a reference frame is fixed on it is a rotating frame with respect to a fixed star frame. Thus a particle at rest or in motion on the earth is acted upon by fictitious forces i.e. Centrifugal and Coriolis forces.

Let us take first Centrifugal force —

The total apparent force of gravity acting on a pendulum is the sum of actual gravitational force and the centrifugal force as given by

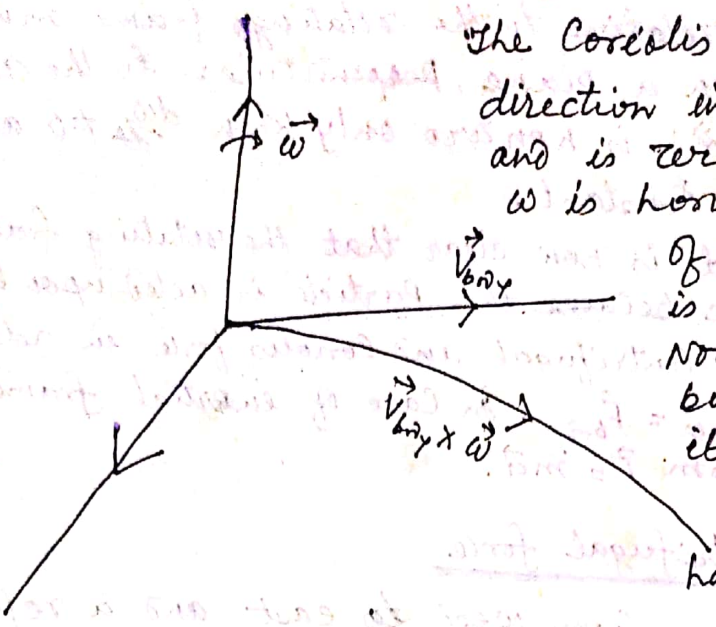
$$g' = g - \vec{\omega} \times (\vec{\omega} \times \vec{r})$$

However the value of g will vary with latitude, being least at equator and greatest at the poles. A plumb line will not point exactly towards the centre of the earth but is swung through a small angle due to the centrifugal force. However the actual measured value differs from theoretical value. This discrepancy is attributed to the fact that earth is not a perfect sphere, and is flattened at the poles. Thus the value of g itself is greater at poles than at the equator, the centrifugal term disregarded. The flattening of earth increases this tendency.

Effect of Coriolis force

The Coriolis force on a moving particle is perpendicular to both $\vec{\omega}$ and $\vec{V}_b$. In the Northern Hemisphere, where $\vec{\omega}$ points out of ground, the Coriolis force $2m(\vec{V}_{body} \times \vec{\omega})$

tends to deflect a Projectile shot along the earth's Surface, to the right of its direction of travel.



The Coriolis deflection reverses direction in the Southern Hemisphere, and is zero at the equator where ω is horizontal. The magnitude of Coriolis acceleration ~~are~~ is always less than 15 cm/sec². Normally it is extremely low but there are instances where it becomes important.

The Coriolis force has deflecting action on the motion of air and water masses on the earth and thereby it affects the weather. The water of rivers in the northern hemisphere which flow along the direction of meridian ie from north to the South or vice-versa, experience a deflection towards the right bank with the Consequence that the right bank of such rivers is steeper than the left bank. The waters of a river that is flowing Southward have a velocity component perpendicular to the axis and directed away from it. If the river flows in a South to north direction, the deflection will be towards the east ie to the right again.

The warm Gulf stream which flows northwards is deflected towards the east, which has a great bearing on the climate of Europe.

Another Practical importance is the occurance of cyclones and tradewinds. whenever a region of low Pressure arises in the northern hemisphere, the air from the surrounding area gets sucked in owing to the Pressure gradient. As the air starts to move, the Coriolis force Causes it to drift the right, Causeing an anticlockwise rotation around the low Pressure zone. This Process Continues till the thrust due to the pressure gradient is balanced by that due to the Coriolis force. This phenomenon Causes cyclones.

It is also responsible for tradewinds. The heating of earth Surface near equator Causes the air to rise by Convection currents and be replaced by Cooler air flowing in towards the equator. The direction of flow is not North-South but due to Coriolis force it gets deviated towards the west. Thus we get the north-west trade winds in Northern hemisphere and similarly south-east trade wind in Southern hemisphere. Effects due to Coriolis force also appear in atomic Physics.