

* Expansion of an ideal gas and changes in thermodynamic properties:-

With the help of 1st law of thermodynamics it is possible to calculate changes in thermodynamic properties such as q , w , ΔU & ΔH when an ideal gas undergoes expansion. The expansion may be isothermal or adiabatic and the process of expansion may be carried out reversibly or irreversibly.

1) Isothermal Expansion:-

a) Calculation of ΔU -

In isothermal process, temperature remains constant. ($T = \text{const}$) and U depends on T . So, $\Delta U = 0$.

b) Calculation of ΔH -

since,

$$H = U + PV$$

Notes

$$\begin{aligned}\Delta H &= \Delta(U + PV) \\ &= \Delta U + \Delta PV \\ &= \Delta U + \Delta nRT\end{aligned}$$

$$\Delta U = 0 \quad \& \quad T = 0.$$

$$\therefore \Delta H = 0.$$

c). calculation of q & w :-

from 1st law of thermodynamics

$$\Delta U = q + w.$$

$$\Delta U = 0.$$

$$\therefore q = -w$$

d). Work done in reversible isother-
mal expansion :-

$$w = -nRT \ln \frac{V_2}{V_1}$$

$$w = -2.303 nRT \log \frac{V_2}{V_1}$$

$$\& \quad q = 2.303 nRT \log \frac{V_2}{V_1}$$

Since, $P_1 V_1 = P_2 V_2$

$$\frac{V_2}{V_1} = \frac{P_1}{P_2}$$

$$W = -2.303 nRT \log \frac{P_1}{P_2}$$

$$q = 2.303 nRT \log \frac{P_1}{P_2}$$

e) Work done in reversible isothermal compression :-

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$$\& \ q = 2.303 nRT \log \frac{P_2}{P_1}$$

2) Adiabatic Expansion :-

In adiabatic expansion, no heat is allowed to enter or leave the system.

Hence, $q = 0$.

From 1st law of thermodynamics -

$$\Delta U = q + w$$

$$q = 0$$

$$\Delta U = w$$

a) Calculation of ΔU :-

We know that, the molar heat capacity at constant volume of an ideal gas is given by -

$$C_V = \left(\frac{\partial U}{\partial T} \right)_V$$

$$\int dU = C_V dT$$

$$\Delta U = C_V \Delta T$$

$$\Delta U = C_V (T_2 - T_1) \quad (\text{for 1-mol})$$

for n -mol.

$$\Delta U = n C_V (T_2 - T_1)$$

b). Calculation of ΔH :-

The molar heat Capacity at constant pressure of an ideal gas is given by -

$$C_p = \left(\frac{\partial H}{\partial T} \right)_p$$

$$\therefore dH = C_p dT$$

$$\therefore \Delta H = C_p \Delta T$$

$$\Delta H = C_p (T_2 - T_1) \quad (\text{for 1-mol})$$

for n-mol

$$\Delta H = nC_p (T_2 - T_1)$$

c). Calculation of w :-

Since, process is adiabatic. So, $q = 0$.

$$\Delta U = w = C_v (T_2 - T_1)$$

$$\Delta U = w = nC_v (T_2 - T_1)$$