Boole's and Weddle's Rule for Numerical Integration:

If we wish to retain differences up to those of the fourth order, we should integrate between x_0 and x_4 and obtain Boole's formula:

$$I = \int_{x_0}^{x_4} y\,dx = \frac{2h}{45}\left[7y_0 + 32y_1 + 12y_2 + 32y_3 + 7y_4\right]$$

or $$\int_{x_0}^{x_n} y\,dx = \frac{2h}{45}\left[7(y_0+y_n) + 32(y_1+y_3+y_5+\cdots) + 12(y_2+y_6+\cdots) + 14(y_4+y_8)\right]$$

and error of this formula can be shown

$$\frac{-8h^7}{945} y^{vi}(\bar{x})$$

If on the other hand, we integrate between x_0 and x_6 retaining differences up to those of the sixth order, we obtaind Weddle's Rule:

$$I = \int_{x_0}^{x_6} y\,dx = \frac{3h}{10}(y_0 + 5y_1 + y_2 + 6y_3 + y_4 + 5y_5 + y_6)$$

and the error in this formula $\left\{-\frac{h^7}{140} y^{vi}(\bar{x})\right\}$

General formula

$$\int y\,dx = \frac{3h}{10}\left[(y_0 + 5y_1 + y_2 + 6y_3 + y_4 + 5y_5 + y_6) + (y_6 + 5y_7 + y_8 + 6y_9 + y_{10} + 5y_{11} + y_{12})\right]$$

Exp. Find the solution using Weddle's Rule for given x and y values in given table

x	0	1	2	3	4	5	6
y	1	0.5	0.2	0.1	0.0588	0.0385	0.027

Solution. Here $a=0$ $b=6$ and $h=1$

Given table for x and y values.

	x	y	
x_0	0	1.0000	$\rightarrow y_0$
x_1	1	0.5000	$\rightarrow y_1$
x_2	2	0.2000	$\rightarrow y_2$
x_3	3	0.1000	$\rightarrow y_3$
x_4	4	0.0588	$\rightarrow y_4$
x_5	5	0.0385	$\rightarrow y_5$
x_6	6	0.0270	$\rightarrow y_6$

Using Weddle's Rule.

$$\int y\,dx = \frac{3h}{10}\left[y_0 + 5y_1 + y_2 + 6y_3 + y_4 + 5y_5 + y_6\right]$$

Putting value of $y_0, y_1 \ldots y_6$ in this formula:

$$\int y\,dx = \frac{3\times1}{10}\left[1 + 5\times.5 + 0.2 + 6\times0.1 + 0.0588 + 5\times0.0385 + 0.027\right]$$

$$= \frac{3}{10}\left[1 + 2.5 + 0.2 + 0.6 + 0.0588 + 0.1925 + 0.027\right]$$

$$= 1.37349$$

Exp. Find solution of $y = \frac{1}{x}$ using Weddle's rule between 1 and 2 and $n=6$

Solution. Here $a=1$, $b=2$ & $n=6$

Then $h = \frac{2-1}{6} = \frac{1}{6}$

Thus value of x and y.

	x	$y = 1/x$
x_0	1	$\frac{1}{1} = 1$
x_1	$1+\frac{1}{6} = \frac{7}{6}$	$\frac{1}{7/6} = 6/7$
x_2	$\frac{7}{6}+\frac{1}{6} = 8/6$	$\frac{1}{8/6} = 6/8$
x_3	$\frac{8}{6}+\frac{1}{6} = 9/6$	$\frac{1}{9/6} = 6/9$
x_4	$\frac{9}{6}+\frac{1}{6} = \frac{10}{6}$	$\frac{1}{10/6} = 6/10$
x_5	$\frac{10}{6}+\frac{1}{6} = 11/6$	$\frac{1}{11/6} = 6/11$
x_6	$\frac{11}{6}+\frac{1}{6} = 12/6$	$\frac{1}{12/6} = 6/12 = 0.5$

Using Weddle's Rule

$$\int y\,dx = \frac{3h}{10}\left[y_0 + 5y_1 + y_2 + 6y_3 + y_4 + 5y_5 + y_6\right]$$

$$= \frac{\cancel{3}\times 1}{10\times \cancel{6}_2}\left[1 + \frac{5\times 6}{7} + \frac{6}{8} + 6\times\frac{6}{9} + \frac{6}{10} + 5\times\frac{6}{11} + 0.5\right]$$

$$= \frac{1}{20}\left[1.5 + 4.286 + 0.75 + 4 + 0.6 + 2.7272\right.$$

$$= \frac{13.863272}{20} = 0.6931$$

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