Exp. Evaluate $p'(3)$ for the polynomial

$$p(x) = x^5 - 6x^4 + 8x^3 + 8x^2 + 4x - 40$$

Solution Here the coefficients are $a_0 = -40$, $a_1 = 4$, $a_2 = 8$, $a_3 = 8$, $a_4 = -6$ and $a_5 = 1$. & $x_0 = 3$

To compute b_0, we form the below table.

$\boxed{b_k = a_k + x_0 b_{k+1}}$ $\qquad b_4 = a_4 + x_0 a_5$

$x_0 = 3$	a_5	a_4	a_3	a_2	a_1	a_0	
	1	-6	8	8	4	-40	
		3	-9	-3	15	57	
	$b_5 = a_n$	b_{n-1}	b_{n-2}	b_{n-2}	b_{n-3}	b_0	
3	1	-3	-1	5	19	17	$= p(3) = b_0$
		3	0	-3	6		
	$c_n = 1$	0	-1	2	25		$= p'(3) = 9$

$\boxed{c_k = b_k + x_0 c_{k+1}}$

Therefore $p'(3) = 25$

Exp Use Birge-Vieta method to find all the positive real roots, rounded off to three decimal places of the equation.

$$x^4 + 7x^3 + 24x^2 + x - 15 = 0$$

Stop the iteration whenever $|x_{i+1} - x_i| < 0.0001$

Solution - We first note that the given equation

$$P_4(x) = x^4 + 7x^3 + 24x^2 + x - 15 = 0 \text{ is of degree 4.}$$

Since $P_4(0) = -15$

$P_4(1) = 19$

By Intermediate value theorem the equation has root lying in]0,1[.

We take $x_0 = 0.5$ as the initial approximation to the root. The first iteration is given by

$$x_1 = x_0 - \frac{P_4(x_0)}{P_4'(x_0)} = 0.5 - \frac{P_4(0.5)}{P_4'(0.5)}$$

Now we evaluate $P_4(0.5)$ and $P_4'(0.5)$ using Horner's method. The results are given in the following table.

	a_4	a_3	a_2	a_1	a_0
	1	7	24	1	-15
0.5		0.5	3.75	13.875	7.4375
	1	7.5	27.75	14.875	$-7.5625 = P_4(0.5)$
0.5		0.5	4.00	15.875	
	1	8.0	31.75	$30.75 = P_4'(0.5)$	

Therefore, $x_1 = 0.5 - \frac{-7.5625}{30.75} = 0.5 + 0.24593$

$x_1 = 0.7459$

The second iteration is given by

$$x_2 = x_1 - \frac{P_4(x_1)}{P_4'(x_1)} = 0.7459 - \frac{P_4(0.7459)}{P_4'(0.7459)}$$

Using synthetic division, we form the following table of values—

Table 2

	1	7	24	1	-15
0.7459		0.7459	5.7777	22.2119	17.3138
	1	7.7459	29.7777	23.2119	$\boxed{2.3138}$ $\rightarrow P_4(0.7459)$
0.7459		0.7459	6.3340	26.9357	
	1	8.4918	36.1117	50.1476	

Therefore $x_2 = 0.7459 - \frac{2.3138}{50.1476} = 0.69976 = 0.6998$

then $x_3 = x_2 - \frac{P_4(x_2)}{P_4'(x_2)} = 0.6998 - \frac{P_4(0.6998)}{P_4'(0.6998)}$

and Table 3.

	1	7	24	1	-15
0.6998		0.6998	5.3881	20.5649	15.0905
	1	7.6998	29.3881	21.5649	$\boxed{0.0905}$
0.6998		0.6998	5.8778	24.6780	
	1	8.3996	35.2659	$\boxed{46.2429}$	

$$x_3 = 0.6998 - \frac{0.0905}{46.2429} = 0.6978$$

for the fourth iteration we have

$$x_4 = x_3 - \frac{P_4(x_3)}{P_4'(x_3)}$$

$$x_4 = 0.6978 - \frac{P_4(0.6978)}{P_4'(0.6978)}$$

and Table 4 for $P_4(0.6978)$ & $P_4'(0.6978)$

Table 4

	1	7	27	1	-15
0.6978		0.6978	5.371525	20.495459	14.999525
	1	7.6978	29.371525	21.495459	$\boxed{0.000475}$
0.6978		0.6978	5.8584497	24.58347	
	1	8.3956	35.2299747	$\boxed{46.078926}$	

$$x_4 = 0.6978 - \frac{0.000475}{46.0789} = 0.6978$$

Since x_3 and x_4 are the same, we get $|x_4 - x_3| < 0.0001$. therefore we stop the iteration here. Hence the approximation value at three decimal places is $\underline{\underline{0.698}}$