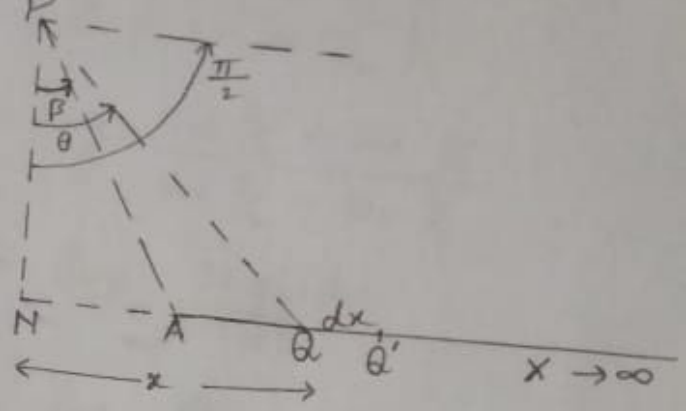
Attraction and Potential

∴ ① ⓔ Find the potential at an external point of a rod infinite at one side.

Let AX ($x \to \infty$) be the given infinite rod with mass per unit length, m



Let P be an external point. Let PN be ⊥ from P to AX. Suppose PN = p.

Let $\angle NPA = \beta$

Take any point Q on the rod and let NQ = x, and $\angle NPQ = \theta$

Then $x = p \tan\theta$

Take an element $QQ' = dx$ of the rod.

Then $QQ' = dx = p \sec^2\theta \, d\theta$

The potential of QQ' at $P = \frac{\gamma m p \sec^2\theta \, d\theta}{p \sec\theta}$

$= \gamma m \sec\theta \, d\theta$

∴ Potential at P of the rod is

$$V = \gamma m \int_{\beta}^{\frac{\pi}{2}} \sec\theta \, d\theta$$

$$= \gamma m \left[\log \tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right)\right]_{\beta}^{\frac{\pi}{2}}$$

$$= c - r_m \log \tan\left(\frac{\pi}{4} + \frac{\beta}{2}\right)$$

where c is an infinite constant

$$= C - r_m \log \left\{\frac{1 + \tan \beta/2}{1 - \tan \beta/2}\right\}$$

$$= C - r_m \log \frac{\cos \beta/2 + \sin \beta/2}{\cos \beta/2 - \sin \beta/2}$$

$$= C - r_m \log \frac{1 + \sin \beta}{\cos \beta}$$

$$= C - r_m \log (\sec \beta + \tan \beta)$$