ATTRACTION AND POTENTIAL

Problems on Potential

Q.) Let V_n be the potential at any point due to a distribution of matter attracting according to the n^{th} power of the distance (and V_{n-2} the potential due to the same distribution attracting as $(n-2)^{th}$ power of the distance), show that

$$\nabla^2 V_n = (n-1)(n+2) V_{n-2}$$

Let m be the mass of the distribution. Let r denote distance when attraction is mr^n then the potential is

$$V_n = \frac{mr^{n+1}}{n+1} \qquad \text{——— (1)}$$

similarly

Now, $\nabla^2 V_n = \nabla \cdot \nabla V_n$

$$= \nabla \cdot \left\{ \frac{m}{n+1} (n+1) r^{n-1} \vec{r} \right\}$$

$\nabla r^n = n r^{n-2} \vec{r}$

$div(\phi \vec{A})$

$= \phi \, div A + \vec{A} \cdot \nabla \phi$

$$= \nabla \cdot \left\{ m r^{n-1} \vec{r} \right\}$$

$$= m \left\{ r^{n-1} div \vec{r} + \vec{r} \cdot \nabla (r^{n-1}) \right\}$$

$$= m \left\{ 3r^{n-1} + \vec{r} (n-1) r^{n-3} \vec{r} \right\}$$

$$= mr^{n-1} \left\{ 3 + n - 1 \right\}$$

$$= V_{n-2} \, (n-1)(n+2)$$

i.e. $\nabla^2 V_n = (n-1)(n+2) V_{n-2}$