ATTRACTION AND POTENTIAL

Q (4) Find the condition that the family of surfaces $\phi(x,y,z) = c$ (where c is a parameter) may be a possible family of equipotential surfaces in free space. (1)

The family of surfaces

$$\phi(x,y,z) = c \quad \text{——} \quad (1)$$

(where c is a parameter) is a family of equipotential surfaces if the potential V at any point (x,y,z) of (1) depends solely on c,

i.e. if there is a functional relation of the form

$$V = f(c) \quad \text{——} \quad (2)$$

between V and c.

(2) gives

$$\frac{\partial V}{\partial x} = f'(c)\frac{\partial c}{\partial x}$$

and $$\frac{\partial^2 V}{\partial x^2} = f''(c)\frac{\partial c}{\partial x} + f(c)\frac{\partial^2 c}{\partial x^2}$$

We have similar expressions for

$$\frac{\partial^2 V}{\partial y^2} \text{ and } \frac{\partial^2 V}{\partial z^2}$$

By Laplace theorem on potential we have

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0 \text{ in free space}$$

$\therefore$ (1) will be a family of equipotential surfaces in free space (2)

if $f''(c)\left\{\left(\frac{\partial c}{\partial x}\right)^2+\left(\frac{\partial c}{\partial y}\right)^2+\left(\frac{\partial c}{\partial z}\right)^2\right\}$

$+ f'(c)\left\{\frac{\partial^2 c}{\partial x^2}+\frac{\partial^2 c}{\partial y^2}+\frac{\partial^2 c}{\partial z^2}\right\}=0$

i.e. if $f''(c)\,(\nabla c)^2 + f'(c)\,\nabla^2(c) = 0$

i.e. if $\frac{\nabla^2(c)}{(\nabla c)^2} = -f''(c)$

$=$ a function $f'(c)$ of c only —— (3)

This is the required condition.

<u>Note</u> :– To find the potential where (1) is a family of equipotential surfaces:-

From (3) we have

$-\frac{f''(c)}{f'(c)}$ = a function of c only

$= \psi(c)$, say

Integrating, we get $\log f'(c) = \log A - \int \psi(c)\,dc$

or, $f'(c) = Ae^{-\int\psi(c)\,dc}$

Integrating again, we get $f(c) = B + A\int e^{-\int\psi(c)\,dc}\,dc$

or, from (2), $V = B + A\int e^{-\int\psi(c)\,dc}\,dc$

where A and B are constants.