Q.) Find the potential of a (solid) sphere at any point.

Let O be the centre and a the radius of the sphere.

Now, any sphere may be regarded as composed of a series of concentric thin spherical shells.

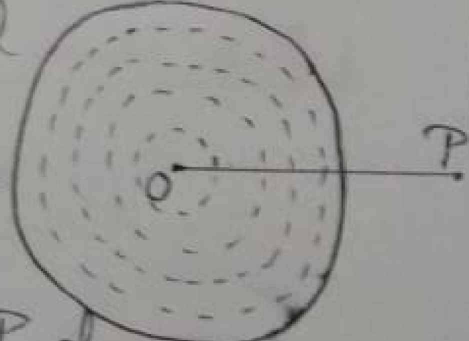
∴ The potential of the sphere at the given point P at which the potential of the sphere is to be determined may be

(i) external to the sphere
(ii) internal to the sphere
or (iii) on the surface of the sphere

Case (i)

Potential at an external point P:-

The point P is external to each of the thin concentric shells comprising the sphere.



The potential at P of each shell

$$= \gamma \frac{\text{mass of the shell}}{OP}$$

$\therefore$ The potential at P of the whole sphere

$$= \frac{\gamma\,(\text{sum of the masses of all the shell})}{OP}$$

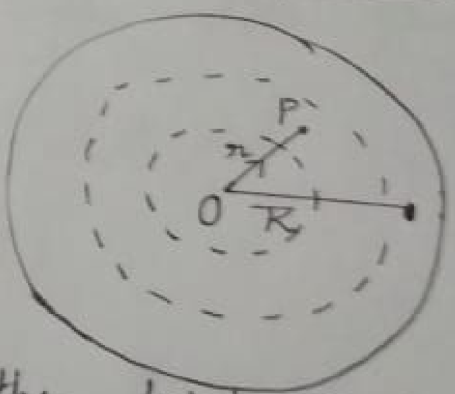
$$= \gamma\,\frac{\text{mass of the sphere}}{OP}$$

$$= \frac{\gamma M}{OP}$$

Case (ii)

Potential at an internal point P

Let the distance of internal point P from the centre O of the sphere be r.



For every concentric shell with radius less than r, the point P is an external point and hence the potential at P due to all such shells

$$= \frac{\gamma\,(\text{total mass of such shells})}{OP}$$

$$= \lim_{\epsilon \to 0} \frac{4\gamma}{3}\,\frac{\rho\pi(r-\epsilon)^3}{r}$$

$$= \frac{4}{3}\gamma\pi r^2\rho$$

where ρ is the density of the uniform sphere.

Now for every concentric shell with the radius R greater than r, the point P is an internal point and the potential at P due to all such shells

$$= \int_r^a \gamma \frac{4\pi R^2 dR}{R} \cdot \rho \quad (a = \text{radius of sphere})$$

$$= \gamma 4\pi\rho \frac{a^2 - r^2}{2}$$

$$= \gamma 2\pi\rho (a^2 - r^2)$$

$\therefore$ Potential at P of the whole sphere is

$$V = \gamma \frac{4}{3}\pi r^2 \rho + \gamma 2\pi\rho (a^2 - r^2)$$

$$= \frac{2}{3}\gamma\pi\rho (3a^2 - r^2)$$

Case (iii)

Potential at a point P on the surface of the sphere

If P be on the surface of the sphere then for every concentric shell P is an external point and hence the potential of the sphere at $P = \frac{M}{a}$

where M is the mass of the sphere.