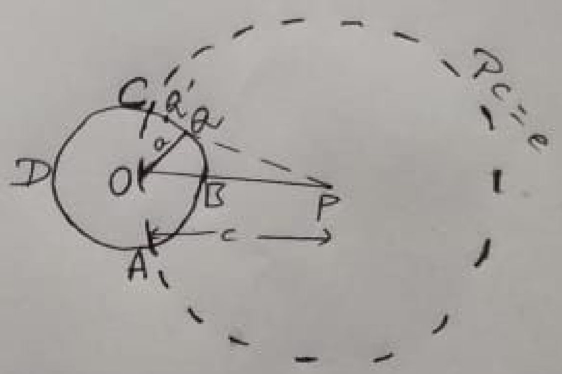
Q. 3 P is a point outside a thin uniform shell at a distance c from the centre. Prove that the sphere of centre P and radius C cuts the shell into two portions whose potential at P are equal.

Let O be the centre and a the radius of the shell.

Let the sphere of centre P and radius C cut the shell into two portions ABC and ADC.



The potential at P due to the portion ABC of the shell is

$$V_1 = \int \frac{\gamma\, 2\pi a \sin\theta \cdot m}{r}$$

$0 \le \theta \le \angle BOC$ (Please see the fig)

m = surf. density

since $r^2 = a^2 + c^2 - 2ac\cos\theta$ so that

$r\,dr = ac\sin\theta\, d\theta$, this gives

$$V_1 = m\gamma \cdot 2\pi a^2 \int_{c-a}^{c} \frac{dr}{dc}$$

$$= \frac{\gamma 4\pi a^2 m}{2c}$$

$$= \frac{\gamma M}{2c}$$ (M = mass of the shell)

similarly potential at P due to the portion CDA of the shell is

$$V_2 = m\gamma \cdot 2\pi a^2 \int_{c}^{c+a} \frac{dr}{dc}$$

$$= \gamma \cdot \frac{4\pi m a^2}{2c}$$

$$= \frac{\gamma M}{2c}$$

clearly $V_1 = V_2$