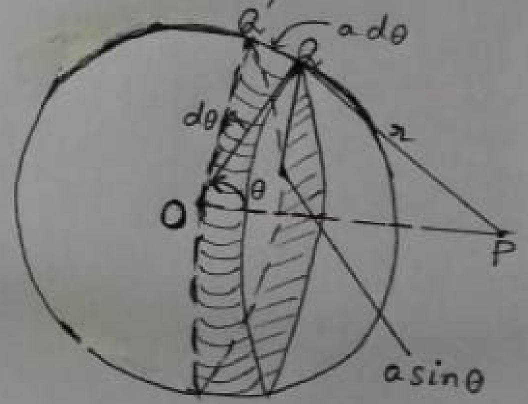
Problems on Potential

Q. 1) Prove that if the law of potential be $\frac{A}{r} e^{-\frac{r}{\lambda}}$, the potential of a uniform thin spherical shell of a radius a at an external point is the same as that of a particle of mass $\frac{\sinh a/\lambda}{a/\lambda}$ times that of the shell placed at the centre.

Let O be the centre of the thin shell and P an external point. Let

$$OP = c$$



Let Q be any point on the surface of the shell with $PQ = r$ and $\angle POQ = \theta$.

The area of a thin zone at Q of the shell the potential at P due to thin zone of the shell

$$= (2\pi a^2 \sin\theta\, d\theta)m \cdot \frac{A}{r} e^{-r/\lambda}$$ where $PQ = r$

$\therefore$ The potential at P of the whole shell is

$$V = 2\pi m a^2 A \int_0^{\pi} \frac{e^{-r/\lambda}}{r} \sin\theta\, d\theta$$

Put $r^2 = a^2 + c^2 - 2ac\cos\theta$

Where $a = OQ$ = radius of the shell.

so that $2r\,dr = 2ac \sin\theta\, d\theta$

$$\therefore V = 2\pi m a^2 A \int_{c-a}^{c+a} \frac{e^{-r/\lambda}}{r} \cdot \frac{r\,dr}{\cancel{a}c}$$

$$= 2\pi m \frac{a}{c} A \int e^{-r/\lambda}\, dr$$

$$= 2\pi m \frac{a}{c} A\lambda \left[e^{-(c-a)/\lambda} - e^{-(c+a)/\lambda}\right]$$

$$= 2\pi m \frac{a}{c} A\lambda \cdot e^{-c/\lambda}\left[e^{a/\lambda} - e^{-a/\lambda}\right]$$

$$= 4\pi m \frac{a}{c} A\lambda e^{-c/\lambda} \sinh \frac{a}{\lambda}$$

$$= 4\pi a^2 m \left(\frac{\lambda}{a} \sinh \frac{a}{\lambda}\right) \frac{A}{c} e^{-c/\lambda}$$

$$= \frac{\sinh a/\lambda}{a/\lambda} (\text{mass of the shell}) \frac{A}{c} e^{-c/\lambda}$$

= Potential at P due to $\left(\frac{\sinh a/\lambda}{a/\lambda}\right)$ times the mass of the shell placed at O.

Note :- (1) The above process may also be applied to find the potential of the shell at an external point under the ordinary law of potential :-

Under the ordinary law of potential the potential at P is

$$V = \int^{\pi} (2\pi a^2 \sin\theta\, d\theta) \frac{m}{r} \quad \text{where } PQ = r$$

③

But $r^2 = a^2 + c^2 - 2ac\cos\theta$

so that $r\,dr = ac\sin\theta\, d\theta$

$$V = 2\pi m a^2 \int_{c-a}^{c+a} \frac{r\,dr}{acr}$$

$$= 2\pi m a^2 (c+a-c+a)$$

$$= \frac{4\pi m a^2}{c}$$

$$= \frac{\text{mass of the shell}}{OP}$$

② The potential at an integral point or the attraction at integral or external point may also be found by this method.