ATTRACTION AND POTENTIAL

(5) Find the attraction of a solid uniform sphere at

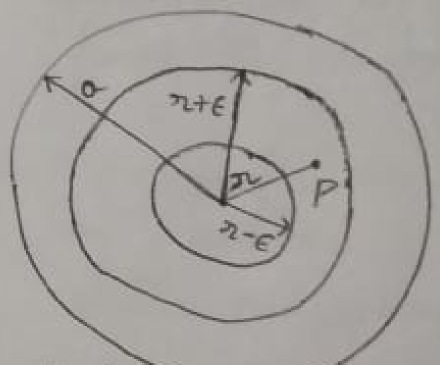
(i) an internal point

(ii) an external point

(iii) a point on its surface

A sphere may be regarded as compound of a series of concentric thin spherical shells.

Let ρ be density, O the centre and a the radius of the uniform solid sphere.



(i) Let P be any internal point at a distance r from the centre O.

Imagine a thin concentric shell of the material of internal radius $r-\epsilon$ and external radius $r+\epsilon$ to be removed from the sphere. Then P is a point in the cavity so formed.

Now we know that the concentric shells external to the cavity exert no attraction at P, and those internal to the cavity attract as if their masses were concentrated at the centre O. Here the attraction of the whole sphere at P.

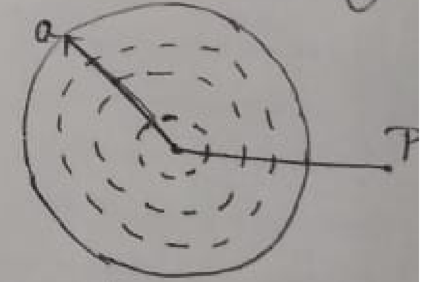
$$= \text{attraction at } P \text{ due to the Concentric shells internal to } P$$

$$= \lim_{\epsilon \to 0} \gamma \cdot \frac{4\pi}{3} \frac{(r-\epsilon)^3}{r^2} \text{ along } PO$$

$$= \gamma \cdot \frac{4\pi}{3} r \text{ along } PO,$$

which is directly proportional to the distance of P from the Centre O

(ii) Let P be a point external to the sphere. Then P is external to each concentric shell comprising the sphere. Hence each Concentric shell attracts P as if its (the shell's) mass were collected at the Centre O of the sphere. Hence the attraction at P due to the sphere, acts along PO and is of magnitude $\frac{\gamma M}{r^2}$ where $OP = r$



(iii) For a point on the surface of the sphere $r = a$.

$\therefore$ using the result either in (i) or in (ii) we see that the attraction at any point P on the surface is $\frac{\gamma M}{a^2}$ along PO.