

Q.) Find the law of density when the attraction at an internal point of a sphere of given mass is constant and equal to its value at the surface.

If a be the radius and M the mass of the sphere, the attraction at the surface is $\frac{\gamma M}{a^2}$. Therefore if $f(R)$ be the density at a distance R from the centre, by question we have

$$\frac{4\pi\gamma}{\pi^2} \int_0^{\pi} f(R) R^2 dR = \frac{\gamma M}{a^2}$$

$$\text{or, } \int_0^{\pi} f(R) \cdot R^2 dR = \frac{M}{4\pi} \frac{\pi^2}{a^2}$$

Differentiating w.r.t. π we get

$$f(\pi) \cdot \pi^2 = \frac{2M}{4\pi} \frac{\pi}{a^2}$$

$$\text{or, } f(\pi) = \frac{M}{2\pi a^2} \cdot \frac{1}{\pi}$$

Thus density is inversely proportional to distance.