ATTRACTION AND POTENTIAL

Q.) Find the potential of a uniform spherical shell at
(i) an internal point
(ii) an external point
(iii) a point on its surface.

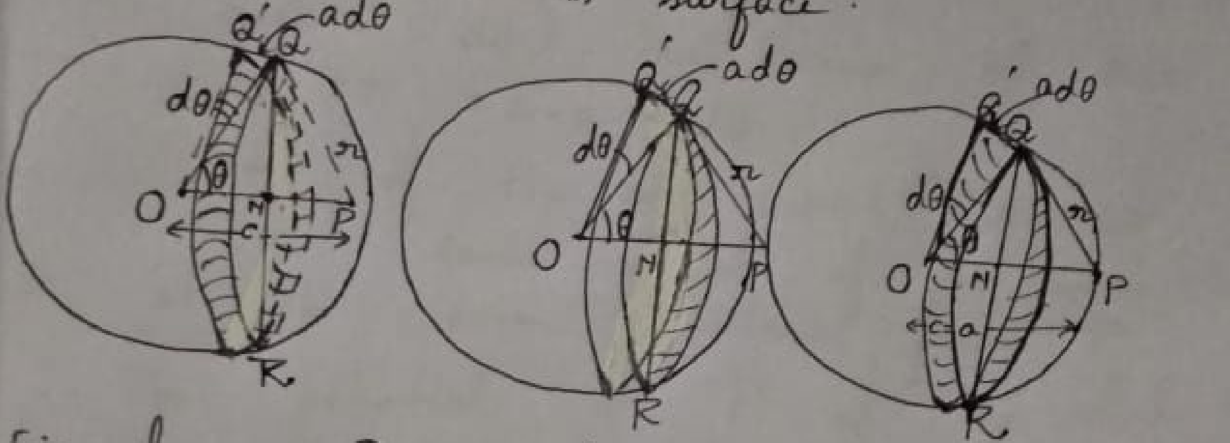

Fig for case (i) Fig for case (ii) Fig for case (iii)

Let a be the radius, O the centre, k the thickness and ρ the density of the material of the thin shell.

Let P be any given point join OP and let $OP = c$, draw a plane perpendicular to OP intersecting the shell in a circle with centre N on OP and radius $NQ = a\sin\theta$, where $\theta = \angle QOP$.

Draw a contiguous parallel plane ($\perp$ OP) intersecting the shell in a contiguous circle through a point Q' adjacent to Q. In this way we get a thin zone of the shell bounded by these two circles. The breadth of this zone is $QQ' = a\,d\theta$ (where $\angle Q'OQ = d\theta$). The area of this zone $= 2\pi \cdot a\sin\theta \cdot a\,d\theta$

$\therefore$ mass of the zone $= 2\pi k \rho a^2 \sin\theta\, d\theta$

Each point of this zone is at the same distance from P.

$\therefore$ The potential at P of the whole zone $= 2\pi k \rho a^2 \sin\theta \cdot \dfrac{d\theta\, \gamma}{PQ}$

$= 2\pi k \rho a^2 \sin\theta\, d\theta \cdot \dfrac{\gamma}{r}$ where $r = PQ$

But $r^2 = a^2 + c^2 - 2ac\cos\theta$ where $OQ = a$
$=$ radius of shell

$\therefore r\,dr = ac\sin\theta\, d\theta$

$\therefore$ The potential at P of the whole zone $= 2\pi \gamma k \rho a \dfrac{dr}{c}$

Case (i) Let P be an internal point. Then r varies from $a-c$ to $a+c$

$\therefore$ The potential at P of the shell is

$$V_i = 2\pi\gamma k\rho\frac{a}{c}\int_{a-c}^{a+c} dr = 4\pi\gamma k\rho a$$

$$= \frac{4\pi a^2 k\rho\cdot\gamma}{a}$$

$$= \frac{\gamma M}{a} \quad \text{where } M = 4\pi a^2 k\rho$$

[Note :— The result shows that the potential of the shell at any internal point is constant and $= \frac{\gamma\,(\text{mass of the shell})}{\text{radius of the shell}}$

Case (ii) Let P be the internal point. Then r varies from $c-a$ to $c+a$

$\therefore$ The potential at P of the shell is

$$V_e = 2\pi\gamma k\rho\frac{a}{c}\int_{c-a}^{c+a} dr = \frac{4\pi\gamma k\rho a^2}{c}$$

$$= \frac{\gamma M}{c}$$

[Note:- Thus the potential at an external point P is the same as that of a particle of mass M, equal to the mass of the shell suppose to be placed at the centre of the shell.

<u>Case (iii)</u> Let P be a point on the surface of the shell. Then $c=a$ and r varies from 0 to $2a$.

$\therefore$ The potential at P of the shell is

$$V_s = 2\pi\gamma k\rho\int_0^{2a} dr = 4\pi k\rho\gamma a$$

$$= \frac{4\pi a^2 k\rho\gamma}{a}$$

$$= \frac{\gamma M}{a}$$

where $M = 4\pi a^2 k\rho$ = mass of the shell

[Note:- Thus the potential at a point P on the surface of the shell is the same as that of a particle of mass M, equal to the mass of the shell, supposed to be placed at the centre of the shell.