Analytical Geometry of two dimensions

Problems (Ellipse) :-

Q.) Find the eccentricity of the ellipse whose lotus rectum is half of the major axis.

Sol:- Let $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ be the equation of the ellipse. Here lotus rectum $= \frac{2b^2}{a}$

major axis $= 2a$

By equation $\frac{2b^2}{a} = \frac{2a}{2} \Rightarrow 2b^2 = a^2$

$$\Rightarrow 2a^2(1-e^2) = a^2 \quad (\because b^2 = a^2(1-e^2))$$

$$\Rightarrow 2(1-e^2) = 1 \Rightarrow 1-e^2 = \frac{1}{2}$$

$$\Rightarrow e^2 = \frac{1}{2}$$

$$\Rightarrow e = \pm \frac{1}{\sqrt{2}}$$

Q.) Find the condition that the line $lx + my + n = 0$ may touch the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Sol:- The equation of the line is

$$lx + my + n = 0 \quad \text{——— (i)}$$

Any tangent to the ellipse is

$$\frac{x \cos\phi}{a} + \frac{y \sin\phi}{b} = 1 \quad \text{——— (ii)}$$

If (i) is a tangent, then Comparing (i) and (ii) we get

$$\frac{\cos\phi}{al} = \frac{\sin\phi}{bm} = \frac{1}{-n}$$

or, $\cos\phi = -\frac{al}{n} = \sin\phi = -\frac{bm}{n}$

Squaring and adding to the eliminate ϕ we get

$$\cos^2\phi + \sin^2\phi = \frac{a^2l^2}{n^2} + \frac{b^2m^2}{n^2} = 1$$

$$= \frac{1}{n^2}(a^2l^2 + b^2m^2)$$

$$\therefore n^2 = a^2l^2 + b^2m^2$$

This is the required Condition.

Q.) If the line $lx + my + n = 0$ touches the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ then show that

$$a^2l^2 + b^2m^2 = n^2$$

Solution: we Know that tangent at the point to the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \text{——— (i)}$$

If the line $lx + my + n = 0$ ——— (ii) touches the ellipse then Comparing (i) and (ii), we get

$$\frac{\cos\phi}{al} = \frac{\sin\phi}{bm} = -\frac{1}{n}$$