Analytical Geometry of two dimensions

Parabola :-

Q.) Find the equation of normal at any point (x_1, y_1) to the parabola.

Sol:- We know that equation of tangent at the point $P(x_1, y_1)$ to the parabola $y^2 = 4ax$ is

$$yy_1 = 2a(x + x_1) \quad \text{—— (i)}$$

$$y = \frac{2a}{y_1}x + \frac{2ax_1}{y_1}$$

Again equation of the line through the point $P(x_1, y_1)$ is

$$y - y_1 = m(x - x_1) \quad \text{—— (ii)}$$

Now equation (ii) be the required equation of normal if it is $\perp$ to (i)

So, by the condition of perpendicularity of the lines

$$m \cdot \frac{2a}{y_1} = -1$$

$$\text{or,} \quad m = \frac{-y}{2a}$$

$\therefore$ from (ii), the required equation of the normal is

$$y - y_1 = \frac{1}{dy/dx}(x - x_1)$$

Q.) Find the Condition that line $y = mx + c$ may be normal to the parabola $y^2 = 4ax$

solution :- we Know that the equation of normal at any point to the parabola $y^2 = 4ax$ is

$$y - y_1 = \frac{-y}{2a}(x - x_1)$$

So the equation of the normal at the point m i.e. at the point $(am^2, 2am)$ will be

$$y + 2am = \frac{2am}{2a}(x - am^2)$$

$$\Rightarrow \quad y = mx - am^3 - 2am \quad \text{——} \ (i)$$

If the line $y = mx + c$ —— (ii) is normal to the parabola then Comparing (i) and (ii) we get

$$c = -am^3 - 2am$$

$$c = -2am - am^3$$

which is the required Condition.