

# Analytical Geometry of two dimensions (1)

## Parabola :-

Q.) Find the equation of normal at any point  $(x_1, y_1)$  to the parabola.

Sol:- We know that equation of tangent at the point  $P(x_1, y_1)$  to the parabola  $y^2 = 4ax$  is

$$yy_1 = 2a(x + x_1) \quad \text{--- (i)}$$

$$y = \frac{2a}{y_1}x + \frac{2ax_1}{y_1}$$

Again equation of the line through the point  $P(x_1, y_1)$  is

$$y - y_1 = m(x - x_1) \quad \text{--- (ii)}$$

Now equation (ii) be the required equation of normal if it is  $\perp$  to (i)

So, by the condition of perpendicularity of the lines

$$m \cdot \frac{2a}{y_1} = -1$$

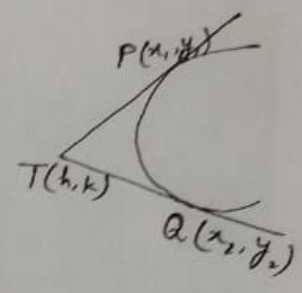
$$\text{or, } m = \frac{-y_1}{2a}$$

$\therefore$  from (ii), the required equation of the normal is

$$y - y_1 = \frac{1}{dy/dx} (x - x_1)$$

Q.) Find the equation of chord of contact at the tangents drawn from an external point  $(h, k)$  to the parabola  $y^2 = 4ax$ .

Sol:- Let TP and TQ be two tangents drawn from an external point  $T(h, k)$  to the parabola  $y^2 = 4ax$ .



Equation of tangents are

$$yy_1 = 2a(x + x_1) \text{ --- (i)}$$

$$y = \frac{2a}{y_1}x + \frac{2ax_1}{y_1} \text{ --- (ii)}$$

Since both the tangents pass through  $T(h, k)$  so its coordinates must satisfy equation (i) and (ii)

∴ we have

$$ky_1 = 2a(h + x_1) \text{ --- (iii)}$$

$$ky_2 = 2a(h + x_2) \text{ --- (iv)}$$

Generalising equation (iii) and (iv) we have

$$ky = 2a(h + x)$$

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This is required equation of the chord of contact.