Analytical Geometry of two dimensions

Problems (PARABOLA):-

Q.1 Find the condition that the line $y = mx + c$ may touch the parabola $y^2 = 4ax$

Sol:- The equation of the parabola is

$$y^2 = 4ax \quad \text{——— (i)}$$

and the equation of the line is $y = mx + c$ ——— (ii)

Eliminating y between (i) and (ii), we get

$$(mx + c)^2 = 4ax$$

$$\Rightarrow m^2x^2 + 2mxc + c^2 = 4ax$$

$$\Rightarrow m^2x^2 + (2mc - 4a)x + c^2 = 0$$

This is a quadratic equation if x. So it will have two roots in order that the line (ii) will become tangent to (i), the root of the above equation must be equal. The condition for this is $D = 0$.

$$\text{i.e. } (2mc - 4a)^2 - 4m^2c^2 = 0$$

$$\Rightarrow 4m^2c^2 - 16mca + 16a^2 - 4m^2c^2 = 0$$

$$\Rightarrow 16a(a - mc) = 0$$

$$\Rightarrow a - mc = 0$$

$$\Rightarrow mc = a$$

$$\Rightarrow c = a/m$$

$\therefore y = mx + \frac{a}{m}$ this is the required equation.

Q.) Find the condition that line $y = mx + c$ may be normal to the parabola $y^2 = 4ax$

<u>solution</u> :- we know that the equation of normal at any point to the parabola $y^2 = 4ax$ is

$$y - y_1 = \frac{-y}{2a}(x - x_1)$$

So the equation of the normal at the point m i.e. at the point $(am^2, 2am)$ will be

$$y + 2am = \frac{2am}{2a}(x - am^2)$$

$$\Rightarrow \quad y = mx - am^3 - 2am \quad \text{——} \quad (i)$$

If the line $y = mx + c$ —— (ii) is normal to the parabola then comparing (i) and (ii) we get

$$c = -am^3 - 2am$$

$$c = -2am - am^3$$

which is the required condition.