Q.) Prove that the locus of centres of circles which intersects two given circles orthogonally is the radical axis of the two circles.

sol:- Let the two given circles be represented by the equation

$$S_1 = x^2+y^2+2g_1x+2f_1y+c_1=0 \quad \text{— (i)}$$

and $$S_2 = x^2+y^2+2f_2x+2f_2y+c_2=0 \quad \text{— (ii)}$$

and let the circle

$$x^2+y^2+2gx+2fy+c=0 \quad \text{— (iii)}$$

Intersect (i) and (ii) orthogonally

Hence $$2gg_1+2ff_1-c-c_1=0 \quad \text{— (iv)}$$

and $$2gg_2+2ff_1-c-c_2=0 \quad \text{— (v)}$$

are the orthogonally conditions

Eliminating c from (iv) and (v) we get

$$2(g_1-g_2)g+2(f_1-f_2)f+c_2-c_1=0$$

obtaining by sbstracting (v) from (iv)

This is the condition satisfied by the Co-ordinates $(-g,-f)$ of the Centre of the circle (iii) Cutting (i) and (ii) orthogonally.

The locus of Centre is then obtained by putting $x=-g$ and $y=-f$ in the Condition.

Hence the required locus is

$$2(g_1 - g_2)x + 2(f_1 - f_2)y + c_2 - c_1 = 0$$

which is the radical axis of (i) and (ii).

Q.) Prove that all circles of a co-axial system are cut orthogonally by every circle passing through the limiting points.

Sol:- Consider a co-axial system of circles given by the equation

$$x^2+y^2+2gx+c=0 \quad \text{——— (i)}$$

where c is the constant and g different for different members of the systems.

The limiting points of the system are $(\sqrt{c}, 0)$ and $(-\sqrt{c}, 0)$. Let the equation of any circle through these limiting points be

$$x^2+y^2+2ux+2vy+k=0 \quad \text{——— (ii)}$$

Since it passes through $(\sqrt{c}, 0)$ and $(-\sqrt{c}, 0)$ we have

$$c+0+2u\sqrt{c}+k=0$$

and $c+0-2u\sqrt{c}+k=0$

$$\therefore 2(c+k)=0 \qquad \therefore k=-c$$

Consequently $u=0$