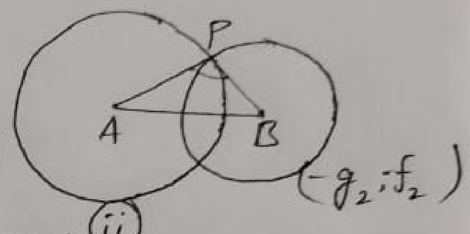
Orthogonal circles

Q.) Define orthogonal circles and find the Condition for orthogonal intersection of two circles.

Sol:- If the two circles intersect at right angles then are said to cut orthogonally.

Let the equation of the two circles be



$$x^2+y^2+2g_1x+2f_1y+c_1=0 \quad \text{— (i)}$$

and $$x^2+y^2+2g_2x+2f_2y+c_2=0 \quad \text{— (ii)}$$

Centre and radius of circle (i) are $(-g_1, -f_1)$ and $\sqrt{g_1^2+f_1^2-c_1}$ respectively

centre and radius of circle (ii) are $(-g_2, -f_2)$ and $\sqrt{g_2^2+f_2^2-c_2}$ respectively.

Since the circle intersect orthogonally, we have $\angle APB = \frac{\pi}{2}$

$$\therefore AB^2 = AP^2 + BP^2 \quad \text{—— (iii)}$$

Now, $AP = \sqrt{g_1^2+f_1^2-c_1}$

$$\therefore AP^2 = g_1^2 + f_1^2 - c_1$$

and $BP = \sqrt{g_1^2+f_1^2-c_2}$

$$\therefore BP^2 = g_1^2 + f_1^2 - c_2$$

ANALYTICAL GEOMETRY OF TWO DIMENSIONS

Q.) Prove that the radical axis of three circles taken in pair are concurrent.

Sol:- Let the equation of the three circles be

$S_1 = 0$ —— (i)

$S_2 = 0$ —— (ii)

and $S_3 = 0$ —— (iii)

Now radical axis of circles (i) and (ii) is

$S_1 - S_2 = 0$ —— (iv)

Again, radical axis of (ii) and (iii) is

$S_2 - S_3 = 0$ —— (v)

And, radical axis of (iii) and (i) circles is

$S_3 - S_1 = 0$ —— (vi)

Adding (iv), (v) and (vi), we get

$S_1 - S_2 + S_2 - S_3 + S_3 - S_1 = 0.$

They are concurrent.