ANALYTICAL GEOMETRY OF TWO DIMENSION

Radical axis :-

Q.) Define radical axis. Find the equation of radical axis of two circles.

Definition :- The radical axis of two circles is the locus of a point which moves such that the lengths of tangents drawn from a point on it to the two circles are equal.

Sol :- Let the two circles be

$$x^2+y^2+2g_1x+2f_1y+c_1=0 \quad \text{—— (i)}$$

and $$x^2+y^2+2g_2x+2f_2y+c_2=0 \quad \text{—— (ii)}$$

length of the tangent drawn from (i)

$$PT_1 = \sqrt{h^2+k^2+2g_1h+2f_1k+c_1}$$

similarly

$$PT_2 = \sqrt{h^2+k^2+2g_2h+2f_2k+c_2}$$

By definition of radical axis,

$$PT_1 = PT_2$$

$$\Rightarrow \sqrt{h^2+k^2+2g_1h+2f_1k+c_1} = \sqrt{h^2+k^2+2g_2h+2f_2k+c_2}$$

squaring, we get

$$h^2+k^2+2g_1h+2f_1k+c_1 = h^2+k^2+2g_2h+2f_2k+c_2$$

i.e. $2(g_1-g_2)h+2(f_1-f_2)k+(c_1-c_2)=0$

Locus of $P(h,k)$ is

$$2(g_1-g_2)x+2(f_1-f_2)y+(c_1-c_2)=0$$

Note :- If the equation of two circles be

$S_1 = 0$ and $S_2 = 0$ then equation of their radical axis will be $S_1 - S_2 = 0$.

Q.2) Prove that the radical axis of two circles is perpendicular to their line of centres.

Sol :- Let $S_1 = x^2 + y^2 + 2g_1 x + 2f_1 y + c_1 = 0$ —— (i)

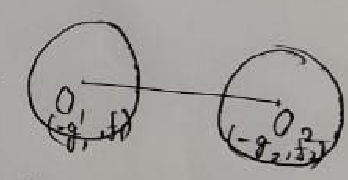
and $S_2 = x^2 + y^2 + 2g_2 x + 2f_2 y + c_2 = 0$ —— (ii)

be two circles whose centres O and O_2 have coordinates $(-g_1, -f_1)$ and $(-g_2, -f_2)$ respectively.

The radical axis of two circles has the equation

$$S_1 - S_2 = 2(g_1 - g_2) + 2(f_1 - f_2) + (c_1 - c_2) = 0$$ —— (iii)

slope m_1 of $O_1 O_2 = \dfrac{f_2 - f_1}{g_2 - g_1}$



m_2 = slope of radical axis

$= -\dfrac{g_1 - g_2}{f_1 - f_2}$

clearly $m_1 m_2 = -1$

Hence radical axis is perpendicular to the line of centres.