Examples — 5

① If the line $2x+3y=1$ touches the parabola $y^2=4ax$ find the length of latus rectum.

sol:- The equation of the parabola

$$y^2=4ax \quad \text{—— ①}$$

Equation of the tangent at the parabola is

$$y=mx+\frac{a}{m} \quad \text{—— ②}$$

For all values of m.

The given line

$$2x+3y=1$$

$$\Rightarrow y=\frac{-2}{3}x+\frac{1}{3} \quad \text{—— ③}$$

touches the parabola ①

Hence equation ② and equation ③ are same

∴ Comparing the coefficients we get

$$\frac{1}{1}=\frac{m}{-2/3}=\frac{a/m}{1/3}$$

$$\therefore m=-2/3$$

and $\frac{a}{3}=\frac{1}{3}$

$$\therefore a=\frac{m}{3}$$

$$=\frac{1}{3}\cdot\frac{-2}{9}=\frac{-2}{9}$$

length of the latus rectum $=4a$

$$=4(-2/a)$$

$$=\frac{8}{9} \text{ (in magnitude)}$$

Ans

(2) Find the locus of the point of interse(?)
of the tangent to the parabola $y^2 = 4ax$. The
angle between them being always a
given angle α.

Sol:- The equation of the any tangent to the
parabola $y^2 - 4ax$ is

$$y = mx = \frac{a}{m}$$

$$\text{or, } y = \frac{m^2x + a}{m}$$

$$\text{or, } m^2x - my + a = 0$$

This is the quadratic equation in m, so we
have two tangents.

Let (h, k) be the point of intersection of
these two tangents then

$$m^2h - mk + a = 0$$

Let the two value of m are m_1 and m_2

$$\therefore m_1 + m_2 = -\frac{-k}{h} = \frac{k}{h}$$

$$\text{and } m_1 m_2 = \frac{a}{h}$$

Then,

$$\tan\alpha = \pm \frac{m_1 - m_2}{1 + m_1 m_2} = \pm \frac{\sqrt{k^2 - 4ah}}{a + h}$$

$$\Rightarrow \tan^2\alpha = \frac{k^2 - 4ah}{(a+h)^2}$$

$$\Rightarrow (a+h)^2 \tan^2\alpha = k^2 - 4ah$$

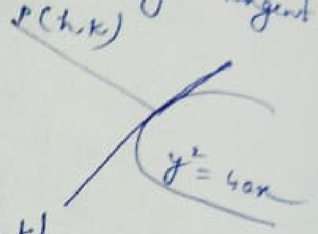
$\therefore$ The locus of (h, k) is

$$(a+x)^2 \tan^2\alpha = y^2 - 4ax$$

3) Find the locus of the foot of perpendicular drawn from a fixed point to any tangent to the parabola.

sol:- Let the equation of the parabola is

$$y^2 = 4ax \quad \text{--- (1)}$$



The fixed point is $P(h,k)$

The equation of the any tangent, to the parabola (1) is

$$y = mx + \frac{a}{m}$$

$$\Rightarrow mx - y + \frac{a}{m} = 0 \quad \text{--- (2)}$$

The equation of any line perpendicular to (2) is

$$x + my + c = 0 \quad \text{--- (3)}$$

Since (3) passes through the point (h,k) then

$$h + mk + c = 0$$

$$\Rightarrow c = -h - mk$$

$\therefore$ From (3) $\quad x + my - h - mk = 0$

$$\Rightarrow m(y-k) = h - x$$

$$\Rightarrow m = \frac{h-x}{y-k}$$

Substituting the value of m in (2) we get

$$\left(\frac{h-x}{y-k}\right)x - y + \frac{a}{\frac{h-x}{y-k}} = 0$$

$$\text{or, } \frac{h-x}{y-k}\cdot x - y + \frac{a(y-k)}{h-x} = 0$$

$$\text{or, } (h-x)^2 x + y(x-h)(y-k) + a(y-k)^2 = 0$$

which is required locus of the foot perpendicular to the tangent.