Analytical geometry of three dimensions

THE PLANE :-

Q.) Prove that a first degree equation represents a plane. or prove that every linear equation in three variables x, y and z always represents a plane.

Ans:- Let the general linear equation in three variables x, y and z be

$$ax + by + cz + d = 0 \quad \text{——— (1)}$$

where a, b, c and d are constants and at least one of a, b and c is not zero.

Let (x_1, y_1, z_1) and (x_2, y_2, z_2) be the co-ordinates of any two points lying on the locus of (1).

Then $$ax_1 + by_1 + cz_1 + d = 0 \quad \text{——— (2)}$$

and $$ax_2 + by_2 + cz_2 + d = 0 \quad \text{——— (3)}$$

Multiplying (2) by m_2 and (3) by m_1 and adding. we get

i.e. $$\frac{a(m_1x_1 + m_2x_1)}{m_1+m_2} + \frac{b(m_1y_2 + m_2y_1)}{m_1+m_2} + \frac{c(m_1z_2 + m_2z_2)}{m_1+m_2} + d = 0$$

when $m_1 + m_2 \neq 0$

This shows that the point

$$\left(\frac{m_1x_2 + m_2x_1}{m_1+m_2}, \frac{m_1y_2 + m_2y_1}{m_1+m_2}, \frac{m_1z_1 + m_2z_1}{m_1+m_2}\right)$$

lies on the locus of (1) for every value of m_1 and m_2.

Thus if the points (x_1, y_1, z_1) and (x_2, y_2, z_2) lie on the locus of (1), then the point dividing them in any ratio $m_1 : m_2$ also lies on the locus of (1).

Hence the line joining (x_1, y_1, z_1) and (x_2, y_2, z_2) completely lies on the focus of (1); in other words, the locus is a plane.

Q.) Find the normal form of the equation of a plane.

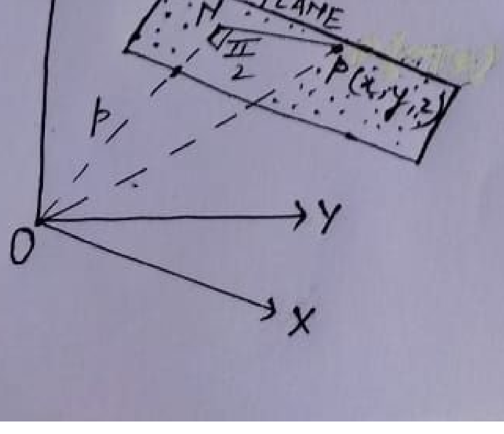
OR

Find the equation of a plane in the form $lx + my + nz = p$, where p is the length of the perpendicular drawn from the origin to the plane and l, m, n are the direction cosines of this normal.

OR

Obtain the equation of a plane, given the length of the normal form the origin and its direction cosines.

Ans:-



Let $OH (= p)$ be the perpendicular from the origin upon a plane. The direction cosines of OH are l, m, n.

Let P be any point (x, y, z) in the plane.

Join M and P and also O and P.

Since OM is normal to the plane and MP lies in the plane, therefore OM is also normal to MP.

$\therefore$ The projection of OP on OM = OM = p.

Also the projection of the line OP joining $O(0,0,0)$ and $P(x, y, z)$ on the line OM whose direction cosines are l, m, n is

$$l(x-0) + m(y-0) + n(z-0), \text{ i.e. } lx + my + nz$$

Hence $lx + my + nz = p$ is the required equation of the plane.