RINGS

A ring R is a set with two binary operations, addition (denoted by $a+b$) and multiplication (denoted by ab) such that for all a, b, c in R:

1. $a+b = b+a$
2. $(a+b)+c = a+(b+c)$
3. There is an additive identity 0. That is, there is an element 0 in R such that $a+0 = a$ for all a in R.
4. There is an element $-a$ in R such that $a+(-a)=0$.
5. $a(bc) = (ab)c$
6. $a(b+c) = ab+ac$ and $(b+c)a = ba+ca$

[A ring is an Abelian group under addition, also having an associative multiplication that is left and right distributive over addition, multiplication need not be commutative. When it is commutative it is called commutative ring] A ring need not have an identity under multiplication. A non zero element of a commutative ring with unity need not have a multiplicative inverse. If it exist we say that it is a unit of the ring. Thus a is a unit of a^{-1} exist.

Examples of Rings :-

Example 1 The set Z of integers under ordinary addition and multiplication is a Commutative ring with unity 1. The units of Z are 1 and -1.

Example 2 :- The set $Z_n = \{0, 1, \ldots, n-1\}$ under addition and multiplication modulo n is a Commutative ring with unity 1. The set of units is $U(n)$.

Example 3 :- The set $Z[x]$ of all polynomials in the variable x with integer coefficients under ordinary addition and multiplication is a Commutative ring with unity $f(x) = 1$.

Example 4 :- The set $M_2(Z)$ of 2×2 matrices with integer entries is a non Commutative ring with unity $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Example 5 :- The set $2Z$ of even integers under ordinary addition and multiplication is a Commutative ring with unity.