CHARACTERISTIC OF A RING.

Let R be a ring with zero element '0' and suppose there exists a positive integer n such that

$$na = a + a + a + \cdots + a \text{ (upto n terms)} = 0$$

$\forall\, a \in R$. Then the smallest such positive integer n is called the characteristic of a ring.

If there does not exist such positive integer n such that $na = 0$ $\forall\, a \in R$ and $a \neq 0$, then the ring R is said to be of characteristic zero or infinite.

<u>For example</u>:- The ring of integers is of characteristic zero and the ring of rational number is also of characteristic zero.

<u>For example</u>:- If $I_6 = \{0, 1, 2, 3, 4, 5\}$. then the ring $\{I_6, +_6, \times_6\}$ is of characteristic 6, because $6a = 0$ $\forall\, a \in I_6$

<u>Theorem</u> :- The characteristic of a ring with unity is zero or $n>0$ according as the unity element 1 regarded as a member of the additive group of the ring has the order zero or n.

<u>Proof</u> :- Let R be a ring with unity element 1 and if order of 1 is zero, then it is obvious that the characteristic of the ring is zero. Let us suppose that the order of 1 is finite say n. then

$$1+1+1+\cdots+1 \ (n \text{ times}) = 0$$

$$n\cdot 1 = 0 \quad \text{———} \ (1)$$

or,

Let a be any non zero element of R, then we have

$$na = a+a+a+\cdots+a \ (n \text{ times})$$

$$= 1\cdot a + 1\cdot a + 1\cdot a + \cdots + 1\cdot a$$

$$(\because R \text{ is with unity})$$

$$= [1+1+1+\cdots+1 \ (n \text{ times})]\cdot a$$

$$= (n\cdot 1)\, a$$

$$= 0\cdot a = 0 \qquad [\text{using } (1)]$$

$\Rightarrow$ order of $a \leq n$.

Hence, the characteristic of R is n.