Rings

Subrings :-

Theorem :- An arbitrary intersection of subrings is a subring.

Proof :- Let $\{S_i : i \in A\}$ be the collection of subrings of a ring R where A is an index set. Then we have to show that $\cap S_i$ is a subring of R.

Let $a, b \in \bigcap_{i \in A} S_i \implies a, b \in S_i \; \forall \; i \in A$.

Since each S_i is a subring of R so we have

$a - b \in S_i \; \forall \; i \in A$ and $ab \in S_i \; \forall \; i \in A$

Therefore we obtain

$a - b \in \bigcap_{i \in A} S_i$ and $ab \in \bigcap_{i \in A} S_i$

Hence $\cap S_i$ is a subring of R.

Corollary :- The union of two subrings is a subring if one is contained in the other.

SUBFIELD :-

Definition :- Let K be a non-empty subset of a field F. Then K is a subfield of F if K itself is a field under the same operations as defined on F.

RINGS

Subring :-

Theorem :- The intersection of the collection of all the subrings containing a given subset M of a ring R is the smallest subring containing M.

Proof :- Let $\{S_\alpha : \alpha \in A\}$ be the collection of all the subrings containing a given subset M of a ring R. That is, $M \subseteq S_\alpha$ for each α. Since R is itself a subring containing M so that the collection S_α is non-empty. Further, the intersection of all subrings containing M is a subring of R containing M. Now we shall have to show that $\cap S_\alpha$ is a smallest subring of R containing M.

Let S be any other subring of R containing M, then obviously $S \subseteq S_\alpha$; therefore, $\cap S_\alpha \subseteq S$. Hence $\cap S_\alpha$ is the smallest subring of R containing M.