RINGS :-

SUBRINGS :- Let S be any nonempty subset of a ring R. Then S is said to be a subring of R if S itself is a ring for the same operations as defined on R.

Improper Subrings :- Let R be a ring. Then $\{0\}$ and R itself are the subrings of R. These subrings are called improper subrings or trivial subrings.

Proper subrings :- Let R be a ring. Then other subrings of R except $\{0\}$ and R called proper subrings.

For examples :-

(i) The ring of rational numbers is a proper subring of the ring of real numbers.

(ii) The ring of even integers is a subring of the ring of integers.

Theorem :- The intersection of two subrings is again a subring.

Proof :- Let S_1 and S_2 be two subrings of a ring R. Then we have to show that $S_1 \cap S_2$ is a subring of R.

For this if $a, b \in S_1 \cap S_2 \Rightarrow a, b \in S_1$ and $a, b \in S_2$ since S_1 and S_2 are subrings of R, then $a-b \in S_1$, $a-b \in S_2$ and and $ab \in S_1$, $ab \in S_2$.

Therefore, we have

If $a-b \in S_1$ and $a-b \in S_2$ and $ab \in S_1$, $ab \in S_2$

Therefore, we have

If $a-b \in S_1$ and $a-b \in S_2$ then $a-b \in S_1 \cap S_2$

and if $ab \in S_1$ and $ab \in S_2$, then $ab \in S_1 \cap S_2$

$\therefore$ $a \in S_1 \cap S_2$, $b \in S_1 \cap S_2 \Rightarrow a-b \in S_1 \cap S_2$;

and $a \in S_1 \cap S_2$, $b \in S_1 \cap S_2 \Rightarrow ab \in S_1 \cap S_2$.

Hence $S_1 \cap S_2$ is a subring of R.