CHARACTERISTIC OF A FIELD

Each element of a field has a certain order finite or infinite, as a member of the additive group of the field.

Definition :- The least positive integer n for which

$$na = 0 \quad , \quad a \in F \text{ (field)}$$

is called characteristic of a field.

Definition :- A field is said to be with characteristic zero or infinity, if the order of the unity as a member of the additive group is infinite. The characteristic is said to be p if the order of unity is finite and equal to p.

Theorem :- The characteristic of a field is either 0 or a prime integer p.

Proof :- We suppose that F is a field and e is the unity of F. The order of e as a member of the additive group of F, may be finite and infinite. If the order of e is infinite, then

$$ne = 0 \iff n = 0$$

Thus the characteristic of F is zero.

If the order of e is finite p (say). Then the characteristic of F is p. We now have to show that p must be prime. Let if possible

$$p = p_1 p_2 \ , \ p_1 \neq 1, \ p_2 \neq 1.$$

We have $0 = pe = (p_1 p_2)e = (p_1 e)(p_2 e)$

$\Rightarrow$ either $p_1 e = 0$ or $p_2 e = 0$

This, however it is impossible for p is the least positive integer such that $pe = 0$. Thus p is prime.

<u>Notes:</u>– Fields with non-zero characteristic are known as modular fields.