# RINGS

## CHARACTERISTIC OF A FIELD :-

Since every field is an integral domain, therefore, the characteristic of a field F is zero or $n > 0$ according as any non-zero element (in particular the unit element 1) of F is order zero or $n$.

In order to find the characteristic of field F, we should find the order of the unit element 1 of F when regarded as a member of the additive group of F. Thus if the order of 1 is zero, then the characteristic of F is zero and if the order of 1 is finite, say $n$, then F is a characteristic of $n$.

For example: The characteristic of the field of real number is 0.

For example :- If $F = \{I_7, '+_7', 'x_7'\}$ be a field, where $I_7 = \{0, 1, 2, 3, 4, 5, 6\}$ then the characteristic of $I_7$ is 7.

Theorem :- Each non-zero element of an integral domain $D$, regarded as a member of the additive group of $D$, is of the same order.

Proof :- Let $a$ be any non-zero element of $D$ and let $o(a) = n$ (say).
We suppose $b$ is any other non-zero element of $D$ and $o(b) = m$ (say).

Since $o(a) = n$

$\Rightarrow na = 0 \quad (\because nx = 0 \ \forall x \in D)$

$\Rightarrow nb = 0$

$\Rightarrow o(b) \leq n$

$\therefore m \leq n$ ———— ①

Also, $o(b) = m$

$\Rightarrow mb = 0$

$\Rightarrow a(mb) = a \cdot 0 = 0$

$\Rightarrow a[b+b+b+ \cdots + b \ (m \text{ times})] = 0$

$\Rightarrow ab + ab + ab + \cdots + ab \ (m \text{ times}) = 0$

$\Rightarrow [a + a + a + \cdots + a \ (m \text{ times})] b = 0$

$\Rightarrow (m\,a) b = 0$

$\Rightarrow ma = 0 \quad (\because b \neq 0$ and $D$ has no zero divisor$)$

$\Rightarrow o(a) \leq m$

$\therefore n \leq m$ ———— ②

From ① and ②, we have
$m = n$

$\Rightarrow o(b) = o(a)$

Also if $o(a)$ is zero, then $o(b)$ will be zero.