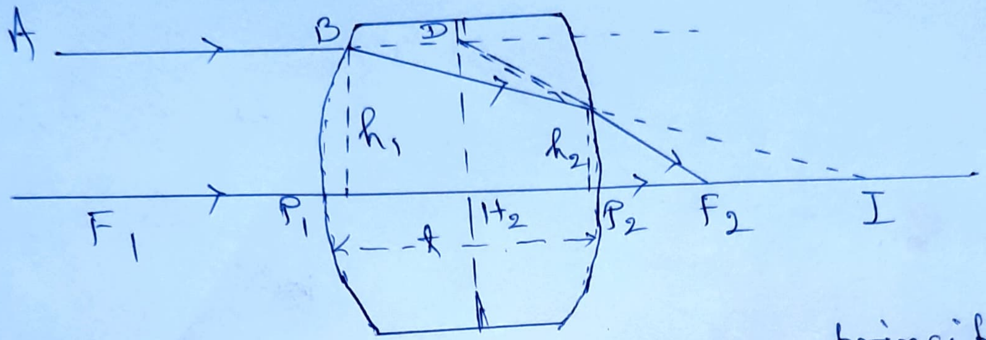
Thick lens formula →

Let us consider a convex lens of thickness t and refractive index μ placed in air. Let R_1 and R_2 be the radii of curvatures of the faces of the lens. The lens is actually a combination of two refracting spherical surfaces with poles P_1 and P_2 as shown in the figure.



Let a ray AB parallel to the principal axis be incident on the first surface at a height h_1 above the principal axis. After refraction from the first spherical surface it follows the paths BC in the lens and meets the second surface at a height h_2 above the axis at point C. This ray, if produced forward, would meet the axis at I, which serves as virtual object for the second surface. After refraction at the second surface, the emergent ray intersects the principal axis at F_2 which is the second focal point of the lens. The incident ray AB produced forward and the emergent ray CF_2 produced backward meet at D. The plane through D and perpendicular to the axis is the second principal plane and its point of intersection with the principal axis H_2 is the second principal point. Thus H_2F_2 is the focal length f of the lens.

Now, for the refraction at a spherical surface the following formula is obeyed

$$\frac{\mu}{v} - \frac{1}{u} = \frac{\mu - 1}{R}$$

Here, for the refraction at the first surface

$u = \infty$, $v = P_1 I$ and $R = R_1$

$$\therefore \frac{\mu}{P_1 I} - \frac{1}{\infty} = \frac{\mu - 1}{R_1}$$

$$\therefore \frac{1}{P_1 I} = \frac{\mu - 1}{\mu R_1} \quad \text{——— (1)}$$

For refraction at the second surface (i.e. from lens to air), we have

$u = P_2 I$, $v = P_2 F_2$, $R = R_2$ and μ will be replaced by $\frac{1}{\mu}$

$$\text{So, } \frac{1/\mu}{P_2 F_2} - \frac{1}{P_2 I} = \frac{\frac{1}{\mu} - 1}{R_2}$$

$$\text{or, } \frac{1}{\mu P_2 F_2} - \frac{1}{P_2 I} = \frac{1-\mu}{\mu R_2}$$

$$\text{or, } \frac{1}{P_2 F_2} - \frac{\mu}{P_2 I} = \frac{1-\mu}{R_2}$$

$$\text{or, } \frac{1}{P_2 F_2} = \frac{\mu}{P_2 I} + \frac{1-\mu}{R_2} \quad \text{——— (2)}$$

From $\Delta^s DF_2H_2$ and $\Delta^s CF_2P_2$, we have

$$\frac{h_1}{h_2} = \frac{H_2 F_2}{P_2 F_2} \quad \text{——— (3)}$$

Similarly for $\Delta^s BIP_1$ and $\Delta^s CIP_2$, we have

$$\frac{h_1}{h_2} = \frac{P_1 I}{P_2 I} \quad \text{——— (4)}$$

Hence from eq^n (3) and (4)

$$\frac{H_2 F_2}{P_2 F_2} = \frac{P_1 I}{P_2 I}$$

$$\therefore \frac{1}{P_2 F_2} = \frac{1}{H_2 F_2} \cdot \frac{P_1 I}{P_2 I} = \frac{1}{f} \frac{P_1 I}{P_2 I} \quad \text{——— (5)}$$

Substituting the value of $\frac{1}{P_2 F_2}$ in eq^n (2), we have

$$\frac{1}{f} \frac{P_1 I}{P_2 I} = \frac{\mu}{P_2 I} + \frac{1-\mu}{R_2}$$

$$\therefore \frac{1}{f} = \frac{\mu}{P_1 I} + \frac{P_2 I}{P_1 I} \cdot \frac{(1-\mu)}{R_2}$$

But from figure, $P_2 I = P_1 I - P_1 P_2 = P_1 I - t$

$$\therefore \frac{1}{f} = \frac{\mu}{P_1 I} + \left(\frac{P_1 I - t}{P_1 I}\right)\left(\frac{1-\mu}{R_2}\right)$$

or, $\frac{1}{f} = \frac{\mu}{P_1 I} + \left(1 - \frac{t}{P_1 I}\right)\left(\frac{1-\mu}{R_2}\right)$

Substituting the value of $\frac{1}{P_1 I}$ from eq^n (1), we get

$$\frac{1}{f} = \mu\left(\frac{\mu-1}{\mu R_1}\right) + \left[1 - \frac{t(\mu-1)}{\mu R_1}\right]\left(\frac{1-\mu}{R_2}\right)$$

or, $\frac{1}{f} = \frac{\mu-1}{R_1} + \frac{1-\mu}{R_2} - \frac{t(\mu-1)(1-\mu)}{\mu R_1 R_2}$

or, $\frac{1}{f} = \frac{\mu-1}{R_1} - \frac{\mu-1}{R_2} + \frac{t(\mu-1)^2}{\mu R_1 R_2}$

or, $\frac{1}{f} = (\mu-1)\left[\frac{1}{R_1} - \frac{1}{R_2} + \frac{t(\mu-1)}{\mu R_1 R_2}\right]$ — (6)

This is called Thick lens formula.

<u>Positions of Cardinal Points</u> →

<u>Second Focal Point</u> → The distance of the second focal point F_2 from the second surface of the lens is P_2F_2.

from eq (5) we have $P_2F_2 = f\left(\frac{P_2 I}{P_1 I}\right) = f\left(\frac{P_1 I - t}{P_1 I}\right) = f\left(1 - \frac{t}{P_1 I}\right)$

But $\frac{1}{P_1 I} = \frac{\mu-1}{\mu R_1}$ [from eq^n (1)]

$\therefore P_2F_2 = +f\left[1 - \frac{t(\mu-1)}{\mu R_1}\right]$ — (7)

<u>Second Principal point</u> → The distance of the second principal point H_2 from the second surface P_2 is

$$\begin{aligned} P_2H_2 &= F_2H_2 - F_2P_2 \\ &= -H_2F_2 + P_2F_2 \\ &= -f + f\left\{1 - \frac{t(\mu-1)}{\mu R_1}\right\} \quad \text{[from eq}^n\ 7] \\ &= -f\frac{(\mu-1)t}{\mu R_1} \quad \text{— (8)} \end{aligned}$$

<u>First Focal Point</u> → If we take the incident ray AB coming from the right, then R_1 and R_2 will interchange and the signs of f, R_1 and R_2 will become opposite. Now, the distance of first focal point F_1 from the first surface P_1 is,

[from eq (7)] $P_1F_1 = -f\left\{1 - \frac{(\mu-1)t}{\mu(-R_2)}\right\}$

$$= -f\left\{1 + \frac{t(\mu-1)}{\mu R_2}\right\} \quad \text{— (9)}$$

First Principal Point → The distance of first principal point (H_1) from the first surface P_1 is

$$P_1H_1 = +f\frac{(\mu-1)t}{\mu(-R_2)} \quad [\text{from eq}^n (8)]$$

$$\therefore P_1H_1 = \frac{-f(\mu-1)t}{\mu R_2} \quad \text{—— (10)}$$

Since the medium on both sides of the lens is same (air), the nodal points N_1 and N_2 will coincide with principal points H_1 and H_2.

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