Abstract Algebra

Binary operation on a set :-

Let G be a non-empty set. Then $G \times G = \{(a,b) : a \in G, b \in G\}$

If $f : G \times G \rightarrow G$, then f is said to be a binary operation on the set G. The image of the orded pair (a,b) under the function f is denoted by $f(a,b)$ or by afb.

Often we use the symbols $+, \times, -, o, *$ etc to denote binary operations on a set. Thus '+' will be binary operation on G iff

$$a+b \in G \ \forall \ a,b \in G \text{ and } a+b \text{ is unique.}$$

Similarly '*' will be binary operation on G iff

$$a*b \in G \ \forall \ a,b \in G \text{ and } a*b \text{ is unique.}$$

A binary operation on a set G is also called a binary Composition in the set G, $a*b$ is a unique element of G. If $a*b \in G \ \forall \ a,b \in G$, then we also say that G is closed with respect to the Composition denoted by $*$.

Examples :- Addition is a binary operation on the set N of natural numbers. The sum of two natural numbers is also a natural number.

Therefore N is closed with respect to addition

i.e. $a+b \in N, \forall a, b \in N$

<u>Subtraction</u> is not a binary operation on N. we have $8-9=-1 \notin N$ whereas $8 \in N, 9 \in N$. Thus N is not closed with respect to subtraction

But subtraction is a binary operation on the set of integers I.

we have $a-b \in I \; \forall a, b \in I$.

<u>Division</u> is not a binary operation in the set R of ~~integers~~ all real numbers. we have $0 \in R$, $5 \in R$ but $5 \div 0$ is not an element of R.

<u>Algebraic structure</u> :-

A non-empty set G equipped with one or more binary operations is called algebraic structure.

If $*$ is a binary operation on G. Then $(G, *)$ is an algebraic structure. $(N, +)$, $(I, +)$, $(I, -)$, $(R, +, \cdot)$ are all algebraic structures.

Obviously addition and multiplication are both binary operations on the set R of real numbers. Therefore $(R, +, \cdot)$ is an algebraic structure equipped with two operations.