

The series of positive terms and the alternating series are special types of these series with arbitrary signs.

(1) If the series of arbitrary signs

$$a_1 + a_2 + a_3 + \dots + a_n + \dots$$

be such that the series

$$|a_1| + |a_2| + |a_3| + \dots + |a_n| + \dots$$

is convergent, then the series  $\sum a_n$  is said to be absolutely convergent.

e.g. A series  $\sum a_n$  is said to be absolutely convergent if the series  $\sum |a_n|$  is convergent. Exp.  $\sum a_n = 1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots$

(2) Conditional Convergence:-

A series  $\sum a_n$  is said to be conditionally convergent if (i)  $\sum a_n$  is convergent and (ii)  $\sum |a_n|$  is divergent (or  $\sum a_n$  is

not absolutely convergent.)

Exp.  $\sum_{n=1}^{\infty} a_n = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \dots$

For Example (Instance)

① The series  $\sum a_n = 1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \frac{1}{5^2} - \dots$

is absolutely convergent, since the series

$$\sum |a_n| = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2} + \dots$$

is known to be convergent.

② The alternating series

$$\sum a_n = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \dots$$

is convergent and the absolute values

$$\sum |a_n| = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots$$

is divergent, so the original

series is conditionally convergent.