Probability

Addition Theorem

Addition Theorem of Probability

- **Addition Theorem**: If two events A and B are mutually exclusive, the probability of occurrence of either A or B is the sum of the individual probability of A and B. Symbolically,
- $P(A \text{ or } B) = P(A) + P(B)$
- The addition theorem is also known as the theorem of total probability.
- **Proof of the Theorem**: If an event A can happen in $\boldsymbol{a_1}$ ways and B can happen in $\boldsymbol{a_2}$ ways, then the number of ways in which either event can happen is $\boldsymbol{a_1 + a_2}$.

Addition Theorem of Probability

- If total number of possible events is **n**, then by definition the probability of either first or the second event happening is

$$\frac{a_1 + a_2}{n} = \frac{a_1}{n} + \frac{a_2}{n}$$

But, $\frac{a_1}{n} = \mathrm{P(A)}$

and, $\frac{a_2}{n} = P(B)$

Hence, P(A or B) = P(A) + P(B),

The theorem can be extended to three or more mutually exclusive events.

Thus, P(A or B or C) = P(A) + P(B) + P(C). **Proved**

Addition Theorem of Probability

- **Example**: A bag contains 30 balls numbered from 1 to 30. One ball is drawn at random, find the probability that the number of the ball will be multiple of 5 or 9.

- **Solution**: Number of multiple of 5 (Event A) = (5, 10, 15, 20, 25 and 30) = 6

Number of multiple of 9 (Event B)= (9, 18, and 27) = 3

Total number of events = 30

$$P(A) = \frac{6}{30}$$

$$P(B) = \frac{3}{30}$$

P(A or B) = P(A) + P(B) (Since the events are mutually exclusive)

$$\frac{6}{30} + \frac{3}{30} = \frac{9}{30} = \mathbf{\frac{3}{10}}\ \boldsymbol{Ans}$$

Addition Theorem of Probability

- **Example**: A person can hit a target in 3 out 4 shots, whereas another person can hit the target in 2 out of 3 shots. Find the probability of the targets being hit at all when they both try.
- **Solution:**

The probability that the first person hit the target = 3/4

The probability that the second person hit the target = 2/3

Addition Theorem of Probability

- The events are **not** mutually exclusive because both of them may hit the target. Hence,

P(A or B) = P(A) + P(B) – P(A and B)

$$= \left(\frac{3}{4} + \frac{2}{3}\right) - \left(\frac{3}{4} \times \frac{2}{3}\right)$$

$$= \frac{17}{12} - \frac{6}{12} = \frac{11}{12} \; Ans$$