Matrices

Inverse of Matrix – Part 2

Inverse of a Matrix

A. Co-factor Method:

- Formula: $A^{-1} = \frac{1}{|A|} . AdjA$

- Where,

- Adj.A is transpose of matrix of cofactors of the elements of matrix A.

Inverse of a Matrix

- **Steps to calculate inverse of matrix:**

i. Calculate the determinant of the matrix; If it is non-zero, then proceed to next step.

ii. Calculate cofactor of each element.

iii. Get matrix of cofactors.

iv. Transposing the matrix of cofactors will give adjoint of A.

v. Apply formula of $A^{-1} = \frac{1}{|A|}.AdjA$

Inverse of a Matrix

❑ **Example 1:** If

$A = \begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$, and I is identity matrix of order 2, then find

i. $B - 4A - 2I$

ii. Calculate A^{-1}

iii. $X \; if \; AX = B$

Solution i: $4A = 4\begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix} = \begin{bmatrix} 8 & -4 \\ 16 & 12 \end{bmatrix}$

$2I = 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

Inverse of a Matrix

$$\therefore B - 4A - 2I = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} - \begin{bmatrix} 8 & -4 \\ 16 & 12 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 - 8 - 2 & 0 + 4 - 0 \\ -2 - 16 - 0 & 1 - 12 - 2 \end{bmatrix}$$

$$= \begin{bmatrix} -9 & 4 \\ -18 & -13 \end{bmatrix}$$ **Answer**

Inverse of a Matrix

(ii) ***Solution***: A^{-1}

$$|A| = \begin{vmatrix} 2 & -1 \\ 4 & 3 \end{vmatrix} = (2 \times 3) - [4 \times (-1)] = 6 + 4 = 10 \neq 0$$

Now,

Cofactor of 2 = 3; Cofactor of -1 = -4

Cofactor of 4 = 1; Cofactor of 3 = 2

Matrix of cofactors = $\begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix}$

- Since adjoint of A is transpose of matrix of cofactors

Inverse of a Matrix

$$AdjA = \begin{bmatrix} 3 & 1 \\ -4 & 2 \end{bmatrix}$$

$$= A^{-1} = \frac{1}{|A|} AdjA$$

$$= A^{-1} = \frac{1}{10}\begin{bmatrix} 3 & 1 \\ -4 & 2 \end{bmatrix}$$

$$= A^{-1} = \begin{bmatrix} 3/10 & 1/10 \\ -4/10 & 2/10 \end{bmatrix}$$ **Answer**

Inverse of a Matrix

(iii) Solution:

$\because\ AX = B$

$\therefore\ \boldsymbol{X = A^{-1}B}$

$$\boldsymbol{X} = \begin{bmatrix} 3/10 & 1/10 \\ -4/10 & 2/10 \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$$

$$X = \begin{bmatrix} \left(\frac{3}{10} \times 1 + \frac{1}{10} \times \langle -2 \rangle\right) & \left(\frac{3}{10} \times 0 + \frac{1}{10} \times \langle 1 \rangle\right) \\ \left(\frac{-4}{10} \times 1 + \frac{2}{10} \times \langle -2 \rangle\right) & \left(\frac{-4}{10} \times 0 + \frac{2}{10} \times \langle 1 \rangle\right) \end{bmatrix}$$

Inverse of a Matrix

$$X = \begin{bmatrix} \left(\frac{1}{10}\right) & \left(\frac{1}{10}\right) \\ \left(\frac{-8}{10}\right) & \left(\frac{2}{10}\right) \end{bmatrix}$$ ***Answer***

Inverse of a Matrix

THANK YOU