

Determinants

Properties based Examples

Properties based Examples

- **Example:**

- Prove that

$$\begin{vmatrix} a + b + 2c & a & b \\ c & b + c + 2a & b \\ c & a & c + a + 2b \end{vmatrix} = 2(a + b + c)^2$$

- **Solution:** Adding column 2&3 in column 1, ($C_1 \rightarrow C_1 + C_2 + C_3$) we get

- $$= \begin{vmatrix} 2a + 2b + 2c & a & b \\ 2a + 2b + 2c & b + c + 2a & b \\ 2a + 2b + 2c & a & c + a + 2b \end{vmatrix}$$

Properties based Examples

- Taking $(2a + 2b + 2c)$ common from column 1

$$\text{■ } (2a + 2b + 2c) \begin{vmatrix} 1 & a & b \\ 1 & b + c + 2a & b \\ 1 & a & c + a + 2b \end{vmatrix}$$

- Now, $R_2 \rightarrow R_2 - R_1$ and $R_3 \rightarrow R_3 - R_1$

$$= 2(a + b + c) \begin{vmatrix} 1 & a & b \\ 0 & b + c + a & 0 \\ 0 & 0 & c + a + b \end{vmatrix}$$

Properties based Examples

$$= 2(a + b + c) \left[1 \begin{vmatrix} a + b + c & 0 \\ 0 & a + b + c \end{vmatrix} + 0 + 0 \right]$$

$$= 2(a + b + c)(a + b + c)(a + b + c)$$

$$= 2(a + b + c)^3 \text{ *Proved*}$$