Determinant

Properties based Examples – 2

Properties based Examples

- **Example 1:** Find the value of $\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$

Solution: Add row 2 and 3 then subtract from row 1:

$$= \begin{vmatrix} (b+c)-(b+c) & a-(2c+a) & a-(a+2b) \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$$

Properties based Examples

$$\begin{vmatrix} 0 & -2c & -2b \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$$

Taking 2 common from row 1, we get

$$-2\begin{vmatrix} 0 & c & b \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$$

Properties based Examples

- Subtracting row 1 from row 2 and row 3,
- $-2\begin{vmatrix} 0 & c & b \\ b & a & 0 \\ c & 0 & a \end{vmatrix}$
- Interchanging column 2 and column 3 (sign of determinant will change)
- $2\begin{vmatrix} 0 & b & c \\ b & 0 & a \\ c & a & 0 \end{vmatrix}$

Properties based Examples

- By expanding the determinant by row 1

$$= 2\left[0 - b\begin{vmatrix} b & a \\ c & 0 \end{vmatrix} + c\begin{vmatrix} b & 0 \\ c & a \end{vmatrix}\right]$$

$$= 2[-b(0 - ac) + c(ab - 0)]$$

$$= 2(abc + abc)$$

$$= 2 \times 2abc$$

$$= \boldsymbol{4abc\ Answer}$$