Determinant

Properties based Examples

Properties based Examples

- **Example 1:** Prove without expanding:

$$\begin{vmatrix} 19 & 17 & 15 \\ 9 & 8 & 7 \\ 1 & 1 & 1 \end{vmatrix} = 0$$

Solution: $C_1 = C_1 - C_3$ and $C_2 = C_2 - C_3$, we get

$$\begin{vmatrix} 19-15 & 17-15 & 15 \\ 9-7 & 8-7 & 7 \\ 1-1 & 1-1 & 1 \end{vmatrix}$$

Properties based Examples

$$= \begin{vmatrix} 4 & 2 & 15 \\ 2 & 1 & 7 \\ 0 & 0 & 1 \end{vmatrix}$$

Taking 2 common from column 1, we get

$$= 2 \begin{vmatrix} 2 & 2 & 15 \\ 1 & 1 & 7 \\ 0 & 0 & 1 \end{vmatrix}$$

Since, column 1 and 2 are same, hence the determinant will be zero. **Proved**

Properties based Examples

- **Example 2:** Following properties of determinant evaluate:

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix}$$

Solution: $C_1 = C_1 - C_3$ and $C_2 = C_2 - C_3$, we get

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix}$$

Properties based Examples

- **Example 1:**

$$\begin{vmatrix} 0 & 0 & 1 \\ a-c & b-c & c \\ a^2-c^2 & b^2-c^2 & c^2 \end{vmatrix}$$

Now, taking (a-c) common from column 1 and (b-c) common from column 2

$$(a-c)(b-c)\begin{vmatrix} 0 & 0 & 1 \\ 1 & 1 & c \\ (a+c) & (b+c) & c^2 \end{vmatrix}$$

Properties based Examples

Expanding the determinant by row 1

$$= (a - c)(b - c)[1(b + c) \; - 1(a + c)]$$

$$= (a - c)(b - c)(b - a)$$

$= (a - b)(b - c)(b - a)$ **Answer**