Determinant

Properties – Part 2

Properties of Determinant

- **Property 5:** If all the elements of any row (or column) is multiplied or divided by any constant (say 'k'), then value of the determinant will also be multiplied or divided by 'k'. This property follows from property 4.
- **Example:**
- $\begin{vmatrix} 2 & 5 \\ 3 & 2 \end{vmatrix}$, *Now multiply the first column by 3, we get*

$$= \begin{vmatrix} 6 & 5 \\ 9 & 2 \end{vmatrix} = 3 \begin{vmatrix} 2 & 5 \\ 3 & 2 \end{vmatrix}$$

Properties of Determinant

- **Property 6:** If any two rows (or column) of any determinant are exactly same, then the value of the determinant will become zero.
- **Example:**
- $\begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & 2 & 3 \end{vmatrix} = 1(3 \times 3 - 1 \times 2) - 2(2 \times 3 - 1 \times 1) + 3(2 \times 2 - 1 \times 3)$
- $(7 - 2 \times 5 + 3 \times 1) = 10 - 10 = 0$

Properties of Determinant

- **Property 7:** If any column (or row) or multiple of any column (or row) is added or subtracted from any other column (or row), value of determinant remains unchanged.
- **Example:**
- $\begin{vmatrix} 1 & 2 & 3 \\ 2 & 4 & 0 \\ 6 & 2 & 12 \end{vmatrix}$, Expanding the determinant by column 3 (C_3), we get
- $3(2 \times 2 - 6 \times 4) - 0 + 12(1 \times 4 - 2 \times 2) = 3 \times (-20) + 12 \times 0 = \mathbf{-60}$

Properties of Determinant

- Multiply column 1 by 2 and subtract from column 2, we get

$$\begin{vmatrix} 1 & 2-2 & 3 \\ 2 & 4-4 & 0 \\ 6 & 2-12 & 12 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 3 \\ 2 & 0 & 0 \\ 6 & -10 & 12 \end{vmatrix}$$, Now expanding the determinant by C_2

$$= 0 + 0 - (-10)(0-6) = \mathbf{-60}$$

Hence, determinant values of pre and post addition or subtraction of column (or row) are same. Proved

Properties of Determinant

- **Property 8:** If determinant is of the form $\begin{vmatrix} a_1+b_1 & c_1 & d_1 \\ a_2+b_2 & c_2 & d_2 \\ a_3+b_3 & c_3 & d_3 \end{vmatrix}$, it can written as sum of two determinant in following form.
- $\begin{vmatrix} a_1 & c_1 & d_1 \\ a_2 & c_2 & d_2 \\ a_3 & c_3 & d_3 \end{vmatrix} + \begin{vmatrix} b_1 & c_1 & d_1 \\ b_2 & c_2 & d_2 \\ b_3 & c_3 & d_3 \end{vmatrix}$

Properties of Determinant

- **Example:** $\begin{vmatrix} 4 & 2 & 1 \\ 7 & 3 & 2 \\ 10 & 4 & 3 \end{vmatrix}$, *By Expanding the determinant by row 1, we get*

$= 4(3\times 3 - 4\times 2) - 2(7\times 3 - 10\times 2) + 1(7\times 4 - 10\times 3) = 4\times 1 - 2\times 1 + 1\times(-2) = \mathbf{0}$,

Now

- $\begin{vmatrix} 4 & 2 & 1 \\ 7 & 3 & 2 \\ 10 & 4 & 3 \end{vmatrix} = \begin{vmatrix} 3+1 & 2 & 1 \\ 5+2 & 3 & 2 \\ 7+3 & 4 & 3 \end{vmatrix} = \begin{vmatrix} 3 & 2 & 1 \\ 5 & 3 & 2 \\ 7 & 4 & 3 \end{vmatrix} + \begin{vmatrix} 1 & 2 & 1 \\ 2 & 3 & 2 \\ 3 & 4 & 3 \end{vmatrix}$

Properties of Determinant

$$\begin{vmatrix} 3 & 2 & 1 \\ 5 & 3 & 2 \\ 7 & 4 & 3 \end{vmatrix} + 0$$ (Since column 1 and column 3 are same, the determinant is zero)

$$\begin{vmatrix} 3 & 2 & 1 \\ 5 & 3 & 2 \\ 7 & 4 & 3 \end{vmatrix}$$, Subtracting column 2 by column88 1, we get

$$\begin{vmatrix} 3-2 & 2 & 1 \\ 5-3 & 3 & 2 \\ 7-4 & 4 & 3 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 1 \\ 2 & 3 & 2 \\ 3 & 4 & 3 \end{vmatrix} = \mathbf{0}$$ (As column 1 and column 3 are same).

- **Hence proved**