Determinants

Properties

Properties of Determinant

- **Property 1:** If all the rows are changed into columns or vice versa, the value of determinant remains unchanged.
- i.e., $|A| = |A'|$, determinant of a matrix and determinant of its transpose are equal.

$$|A| = \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = 1 \times 4 \ - 2 \times 3 = 4 - 6 = -2$$

$$|A'| = \begin{vmatrix} 1 & 3 \\ 2 & 4 \end{vmatrix} = 1 \times 4 \ - 2 \times 3 = 4 - 6 = -2$$

Hence, $|A| = |A'|$ Proved

Properties of Determinant

- **Property 2:** If any two (or column) are interchanged, the sign of determinant changes but absolute value remains same.

- Example: $\begin{vmatrix} 1 & 3 & 0 \\ 2 & 4 & 0 \\ 1 & 0 & 3 \end{vmatrix} = (-)\begin{vmatrix} 3 & 1 & 0 \\ 4 & 2 & 0 \\ 0 & 1 & 3 \end{vmatrix}$ [by interchanging column 1 and 2].

- Similarly, $\begin{vmatrix} 1 & 3 & 0 \\ 2 & 4 & 0 \\ 1 & 0 & 3 \end{vmatrix} = (-)\begin{vmatrix} 2 & 4 & 0 \\ 1 & 3 & 0 \\ 1 & 0 & 3 \end{vmatrix}$ [by interchanging row 1 and 2].

Properties of Determinant

- **Property 3:** If any column (or row) is shifted over any adjacent column (or row), then the absolute value of the determinant does not change but sign changes if the jumped columns (or row) are odd in number.
- Example: $\begin{vmatrix} 2 & 1 & -2 & 2 \\ 4 & 3 & 1 & 1 \\ 0 & 2 & 0 & 1 \\ 3 & 4 & 3 & 1 \end{vmatrix} = (-)^2 \begin{vmatrix} 1 & -2 & 2 & 2 \\ 3 & 1 & 4 & 1 \\ 2 & 0 & 0 & 1 \\ 4 & 3 & 3 & 1 \end{vmatrix}$
- Sign does not changes because number of jumped columns are two in number (even number).

Properties of Determinant

- $$\begin{vmatrix} 2 & 1 & -2 & 2 \\ 4 & 3 & 1 & 1 \\ 0 & 2 & 0 & 1 \\ 3 & 4 & 3 & 1 \end{vmatrix} = (-)^3 \begin{vmatrix} 1 & -2 & 2 & 2 \\ 3 & 1 & 1 & 4 \\ 2 & 0 & 1 & 0 \\ 4 & 3 & 1 & 3 \end{vmatrix}$$
- Sign does change because number of jumped columns are three in number (odd number).

Properties of Determinant

- **Property 4:** If there is any common factor in the elements of any row (or column), it can be taken out.
- Example: $\begin{vmatrix} 3 & 1 & 0 \\ 6 & 2 & 1 \\ 9 & 1 & 3 \end{vmatrix}$
- Since, 3 is common in all elements of first column, 3 can be taken out.

Hence, $\begin{vmatrix} 3 & 1 & 0 \\ 6 & 2 & 1 \\ 9 & 1 & 3 \end{vmatrix} = (3) \begin{vmatrix} 1 & 1 & 0 \\ 2 & 2 & 1 \\ 3 & 1 & 3 \end{vmatrix}$