Matrix Algebra

Determinant

Matrices and Determinants

- **Determinant:** To each square matrix corresponds a determinant. A single value associated with the a square matrix or is calculated from a square matrix is called determinant.
- **Value of a Determinant:**
- The third order determinant $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$

Matrices and Determinants

- *We find the determinant by expanding the determinant by any row or any column.*

- *For example, suppose we expand by row 1 of the above square matrix, then*
- $D = a_1A_{11} + b_1A_{12} + c_1A_{13}$
- Where a_1, b_1, c_1 are the elements of row 1; and A_{11}, A_{12}, A_{13} are corresponding cofactors of a_1, b_1, c_1.
- If we *expand by column 1 of the above square matrix, then*
- $D = a_1A_{11} + a_2A_{21} + a_3A_{31}$
- If we *expand by 2nd row of the above square matrix, then*
- $D = a_2A_{21} + b_2A_{22} + c_3A_{23}$ ---- *and so on.*

Matrices and Determinants

- **Cofactor** of an element is the minor of that element multiplied by $(-1)^{i+j}$.
- **Minor** of an element is lower order determinant of obtained by deleting the row and column of the element.
- For example:
- The minor of a_1 of the determinant $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ is $= \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix}$;

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- Similarly, minor of b_2 of the determinant $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ is $= \begin{vmatrix} a_1 & c_1 \\ a_3 & c_3 \end{vmatrix}$;

  and so on

- There will as many minors and cofactors as are number of elements in the matrix. In 2×2 matrix, there will be 4 minors and cofactor corresponding to each element. Similarly, in 3×3 matrix, there will be 9 minors and 9 cofactors corresponding to each element.

Matrices and Determinants

- **Example 1:** $\begin{vmatrix} 3 & 4 \\ 7 & -2 \end{vmatrix}$
- The value of the determinant = $(3) \times (-2) + (-4) \times (10) = -6 - 40 = -46\ Ans$
- **Example 2:** $\begin{vmatrix} \mathbf{3} & \mathbf{4} & \mathbf{7} \\ \mathbf{2} & \mathbf{1} & \mathbf{3} \\ \mathbf{7} & \mathbf{2} & \mathbf{1} \end{vmatrix}$
- *By expanding the determinant by row 1 we get*
- $D = 3A_{11} + 4A_{12} + 7A_{13}$

Matrices and Determinants

$$= (3)\begin{vmatrix} 1 & 3 \\ 2 & 1 \end{vmatrix} + (-1)^{(1+2)}(4)\begin{vmatrix} 2 & 3 \\ 7 & 1 \end{vmatrix} + (-1)^{(1+3)}(7)\begin{vmatrix} 2 & 1 \\ 7 & 2 \end{vmatrix}$$

$$= (3) \times (1 - 6) - (4) \times (2 - 21) + (7) \times (4 - 7)$$

$$= (3)(-5) - (4)(-19) + (7) \times (-3)$$

$$-15 + 76 - 21 = -40\ Ans$$