Probability

Multiplication Theorem

Multiplication Theorem of Probability

- **Multiplication Theorem**: This theorem states that if two events A and B are independent the probability that they both will occur is equal to product of probability of their individual probabilities. Symbolically, if A and B are independent, then

  P(A and B) = P(A) × P(B).

  The theorem can be extended to three or more independent events. Thus,

  P(A and B) = P(A) × P(B) × P(B).

Multiplication Theorem of Probability

- **Proof of the Theorem**:
- If an event A can happen in n_1 of which a_1 are successful ways and event B can happen in n_2 of which a_2 are successful ways, we can combine each successful event in the first with each successful event in the second case. Thus the total number of successful events in the both the cases is $a_1 \times a_2$. Similarly total number of possible cases is $n_1 \times n_2$.

Multiplication Theorem of Probability

- Then by definition, the probability of bot independent events happening is

$$\frac{a_1 \times a_2}{n_1 \times n_2} = \frac{a_1}{n_1} \times \frac{a_2}{n_2}$$

But, $\frac{a_1}{n_1}$ = P(A), and

$\frac{a_2}{n_2}$ = P(B).

$$\therefore \quad P(A \text{ and } B) = P(A) \times P(B) = \frac{a_1}{n_1} \times \frac{a_2}{n_2}$$

In similar way, theorem can be extended to three or more events.

Multiplication Theorem of Probability

- **Example**: A bag contains 5 white and 3 black balls. Two balls are drawn at random one after another without replacement. Find the probability that

  (i) both are black; (ii) one black and one white.

- **Solution**:

(i) Probability of drawing a black ball in the first attempt $= \frac{3}{3+5} = \frac{3}{8}$

Probability of drawing a black ball in the second attempt $= \frac{2}{2+5} = \frac{2}{7}$

Probability that both balls drawn are black

$$P(A \text{ and } B) = P(A) \times P(B)$$

$$= \frac{3}{8} \times \frac{2}{7} = \mathbf{\frac{3}{28}}\ \textbf{\textit{Ans}}$$

Multiplication Theorem of Probability

- **Case I:** Probability of drawing a black ball in the first attempt $= \frac{3}{3+5} = \frac{3}{8}$

Probability of drawing a white ball in the second attempt $= \frac{5}{2+5} = \frac{5}{7}$

Probability that first ball is black and second is white

$$P(A \text{ and } B) = P(A) \times P(B)$$

$$= \frac{3}{8} \times \frac{5}{7} = \mathbf{\frac{15}{56}}\ \textbf{\textit{Ans}}$$

Multiplication Theorem of Probability

- **Case II:** Probability of drawing a white ball in the first attempt $= \frac{5}{3+5} = \frac{5}{8}$

Probability of drawing a black ball in the second attempt $= \frac{3}{3+4} = \frac{3}{7}$

Probability that first ball drawn is white and second black

$$P(A \text{ and } B) = P(A) \times P(B)$$

$$= \frac{5}{8} \times \frac{3}{7} = \mathbf{\frac{15}{56}}$$

One ball is white and another black can happen in either way Case I or Case II.

Hence, probability that one ball is white and one is black $= \left(\frac{15}{56} + \frac{15}{56}\right) = \frac{30}{56} = \mathbf{\frac{15}{28}}$ **Ans**