Equation of the normal - The normal at any point (x, y) is that straight line which is perpendicular to the tangent at (x, y).

Thus to find out the equation of the normal at the point (x, y).
We shall have to find out the equation of that straight line which passes through (x, y) and is perpendicular to the tangent at (x, y).
We therefore assume that the equation of the normal is

$$Y - y = m(X - x) \quad — (1)$$

We know the equation of tangent at the point (x, y) is

$$Y - y = \frac{dy}{dx}(X - x) \quad — (11)$$

From (1) & (11) are perpendicular to each other then

$$m \times \frac{dy}{dx} = -1$$

i.e $\quad m = -\dfrac{1}{dy/dx}$

putting m in Eqn (1) we get

$$Y - y = -\frac{1}{dy/dx}(X - x)$$

$$(Y - y)\frac{dy}{dx} = -(X - x)$$

$$\Rightarrow (Y - y)\frac{dy}{dx} + (X - x) \text{\_}$$ This is Eqn of normal.

Exp. Find the epuction of the tangent and normal to the curve $x^{2/3} + y^{2/3} = a^{2/3}$ at the point θ.

Sol. Let $P(x = a\cos^3\theta,\ y = a\sin^3\theta)$ be any point on the given curve.

Then $\frac{dy}{d\theta} = a \cdot 3 \cdot \sin^2\theta \cdot \cos\theta$

$$\frac{dx}{d\theta} = a \cdot 3 \cdot \cos^2\theta \cdot (-\sin\theta)$$

$$\frac{dy}{dx} = \frac{dy}{d\theta} \cdot \frac{d\theta}{dx} = \frac{dy}{d\theta} \Big/ \frac{dx}{d\theta}$$

$$\frac{dy}{dx} = \frac{\cancel{3a}\ \sin^{\cancel{2}}\theta \cdot \cancel{\cos\theta}}{\cancel{3a}\ \cos^{\cancel{2}}\theta\ (-\cancel{\sin\theta})} = -\tan\theta$$

$$m = -1\Big/\frac{dy}{dx} = \frac{-1}{\frac{dy}{dx}} = \frac{-1}{-\tan\theta} = \cot\theta$$

Then the epuation of tangent at $P(x = a\cos^3\theta,\ y = a\sin^3\theta)$ is

$$(y - a\sin^3\theta) = (-\tan\theta)(x - a\cos^3\theta)$$

$$y - a\sin^3\theta = -\frac{\sin\theta}{\cos\theta}(x - a\cos^3\theta$$

$$\cos\theta y - a\cos\theta\sin^3\theta = -\sin\theta x + a\sin\theta\cos^3\theta$$

$$\sin\theta x + \cos\theta y = a\cos\theta\sin^3\theta + a\sin\theta\cos^3\theta$$

$$= a\sin\theta\cos\theta(\sin^2\theta + \cos^2\theta)$$

$$\underline{\sin\theta x + \cos\theta y = a\sin\theta\cos\theta}$$ This is tangent

Now the equation of Normal

$$(Y-y)\frac{dy}{dx} + (X-x) = 0$$

$$(y - a\sin^3\theta)(-\tan\theta) + (x - a\cos^3\theta) = 0$$

$$(y - a\sin^3\theta)\left(-\frac{\sin\theta}{\cos\theta}\right) + x - a\cos^3\theta = 0$$

$$(-\sin\theta)(y - a\sin^3\theta) + \cos\theta(x - a\cos^3\theta) = 0$$

$$-\sin\theta\, y + a\sin^4\theta + \cos\theta\, x - a\cos^4\theta = 0$$

$$\cos\theta\, x - \sin\theta\, y + a(\sin^4\theta - \cos^4\theta) = 0$$

$$\cos\theta\, x - \sin\theta\, y + a[\underbrace{(\sin^2\theta + \cos^2\theta)}_{\text{is } 1}(\sin^2\theta - \cos^2\theta)] = 0$$

$$\cos\theta\, x - \sin\theta\, y + a(\sin^2\theta - \cos^2\theta) = 0$$

$$\cos\theta\, x - \sin\theta\, y \cancel{+} a(2\sin^2\theta - 1) = 0$$

$$\cos\theta\, x - \sin\theta\, y + a(-\cos 2\theta) = 0$$

$\because \cos 2\theta = 1 - 2\sin^2\theta$

$$\cos\theta\, x - \sin\theta\, y = a\cos 2\theta$$