Muller's Method :-

In this method, the given function f(x) is approximated by a second degree curve in the vicinity of a root. The roots of the quadratic are then assumed to be the approximations to the roots of the epuations f(x)=0. The method is iterative and can be used to compute complex roots. It has quadratic convergence.

Let (x_{i-2}, y_{i-2}), (x_{i-1}, y_{i-1}) and (x_i, y_i) be three distinct points on the curve $y=f(x)$ where x_{i-2}, x_{i-1} and x_i are approximations to a root of $f(x)=0$. How, a second degree curve passing through the three points is given Lagrange's formula -

$$L(x) = \frac{(x-x_{i-1})(x-x_i)}{(x_{i-2}-x_{i-1})(x_{i-2}-x_i)} y_{i-2} + \frac{(x-x_{i-2})(x-x_i)}{(x_{i-1}-x_{i-2})(x_{i-1}-x_i)} y_{i-1}$$

$$+ \frac{(x-x_{i-2})(x-x_{i-1})}{(x_i-x_{i-2})(x_i-x_{i-1})} y_i \quad \text{—— (1)}$$

Let $h_i = x_i - x_{i-1}$, $h_{i-1} = x_{i-1} - x_{i-2}$ —— (2)

Then

$$\left.\begin{aligned} x-x_{i-1} &= x-x_i+x_i-x_{i-1} = (x-x_i)+h_i \\ x-x_{i-2} &= x-x_i+x_i-x_{i-2} = (x-x_i)+(h_{i-1}+h_i) \\ x_{i-2}-x_{i-1} &= -h_{i-1} \\ x_{i-2}-x_i &= -(h_{i-1}+h_i) \text{ and } \Delta_i = y_i - y_{i-1} \end{aligned}\right\} \text{—— (3)}$$

Hence

$$L(x) = \frac{(x-x_i+h_i)(x-x_i)}{h_{i-1}(h_{i-1}+h_i)} y_{i-2} + \frac{(x-x_i+h_{i-1}+h_i)(x-x_i)}{-h_{i-1}h_i} y_{i-1}$$

$$+ \frac{(x-x_i+h_i+h_{i-1})(x-x_i+h_i)}{h_i(h_{i-1}+h_i)} y_i \quad \text{—— (4)}$$

After simplification, the preceding equation can be written as

$$L(x) = A(x-x_i)^2 + B(x-x_i) + y_i$$

where $A = \frac{1}{(h_{i-1}+h_i)}\left(\frac{\Delta_i}{h_i} - \frac{\Delta_{i-1}}{h_{i-1}}\right)$

and $B = \frac{\Delta_i}{h_i} + A h_i$ —— (5)

With these values of A and B, the quadratic equation (1) gives the next approximation x_{i+1}

$$x_{i+1} = x_i + \frac{-B \pm \sqrt{B^2 - 4Ay_i}}{2A} \quad \text{—— (6)}$$

Since Eqn (6) leads to inaccurate results, we take the the equivalent form

$$x_{i+1} = x_i - \frac{2y_i}{B \pm \sqrt{B^2 - 4Ay_i}} \quad \text{—— (7)}$$

This Equation (7) gives the next approximation to the root. In eqn. (7), the sign in the denominator should be chosen so that the denominator will be largest in magnitude.