Maxwell- Boltzmann Statistics -

The Maxwell-Boltzmann statistics is applicable to a system whose particles are identical and distinguishable.

Let us ~~consid~~ consider a system of N- identical (non-interacting) particles. For simplicity particles are assumed to be spinless. An energy value of an ideal system is a sum of single particle called levels. These are given by,

$$\epsilon_p = \frac{p^2}{2m} \quad \text{——} \quad (1)$$

where 'm' is a mass of a particle and $p = |\vec{p}|$, $\vec{p}$ being the momentum of a single particle.

Let n_p be the number of particles in a gas which are in a given quantum state. The number n_p are sometime called occupation numbers of the various quantum states. A state of an ideal system can be specified by a set of occupation number $\{n_p\}$. Hence the total number of particles N and total energy E of the system are given by,

$$N = \sum_p n_p \quad \text{——} \quad (2)$$

$$E = \sum_p n_p \epsilon_p \quad \text{——} \quad (3)$$

where $n_p = 0, 1, 2 \ldots$

In the thermodynamics

Limit $v \to \infty$, the value of $\vec{p}$ becomes a continuum. Then the energy E forms a continuum spectrum. Let us divide the energy spectrum of (3) into group of states, each group containing respectively $g_1, g_2, g_3 \ldots$ substates as shown in fig. Each group is called cell and has an average energy (ϵ_i) and numbers of particles n_i such that

$$N = \sum_i n_i = \text{constant} \quad \text{— (4)}$$

$$E = \sum_i n_i \epsilon_i = \text{constant} \quad \text{— (5)}.$$

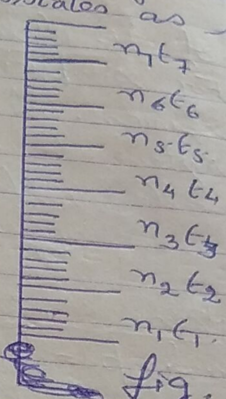


fig.

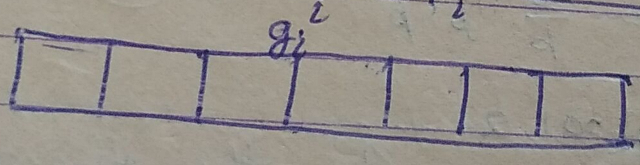
Since the total number of particles N and total energy E are constant, we have from (4) & (5)

$$\delta N = \sum_i \delta n_i = 0. \quad \text{— (6)}$$

$$\delta E = \sum_i \epsilon_i \delta n_i = 0 \quad \text{— (7)}$$

In order of calculate entropy of the gas, we calculate the statistical weight $\Delta\Gamma$ of the system. Regarding each cell of n_i particles as an independent subsystem, whose statistical weight is $\Delta\Gamma_i$, we can write

$$\Delta\Gamma = \prod_i \Delta\Gamma_i \quad \text{— (8)}$$



which is known as variational principle or Hamilton's variational principle or simply Hamilton's principle.

The variation of L is given by,

$$\delta L = L(q+\delta q, \dot{q}+\delta\dot{q}, t) - L(q,\dot{q},t) \quad \text{---(3)}$$

$$f(a+x) = f(a) + \left(\frac{\delta f}{\partial x}\right)_a + \frac{1}{2}\left(\frac{\partial^2 f}{\partial x^2}\right)x^2 + \cdots$$

$$f(a+x, b+y) = f(a,b) + \left(\frac{\partial f}{\partial x}\right)x + \left(\frac{\partial f}{\partial y}\right)y + \cdots$$

Expanding the powers of δq and $\delta\dot{q}$, we have the first order term.

$$\delta L = \cancel{L(q,\dot{q},t)} + \frac{\partial L}{\partial q}\delta q + \frac{\partial L}{\partial \dot{q}}\delta\dot{q} - \cancel{L(q,\dot{q},t)}$$

$$= \frac{\partial L}{\partial q}\delta q + \frac{\partial L}{\partial \dot{q}}\delta\dot{q} \quad \text{——— (4)}$$

Where δq and $\delta\dot{q}$ are called the variations of $q(t)$ and $\dot{q}(t)$ respectively. They are finite everywhere between t_1 and t_2 except at the two extremes where they are zero i.e. $\delta q(t_1) = \delta q(t_2) = 0$

Substituting (4) in (2) we get,

$$\delta S = \int_{t_1}^{t_2}\left[\frac{\partial L}{\partial q}\delta q + \frac{\partial L}{\partial \dot{q}}\delta\dot{q}\right]dt$$

$$= \int_{t_1}^{t_2}\frac{\partial L}{\partial q}\delta q\, dt + \int_{t_1}^{t_2}\frac{\partial L}{\partial \dot{q}}\delta\dot{q}\, dt \quad \text{----(5)}$$

Since, $\delta\dot{q} = \delta\frac{dq}{dt} = \frac{d(q+\delta q)}{dt} - \frac{dq}{dt}$

$$= \frac{d}{dt}\left[(q+\delta q) - q\right]$$

$$= \frac{d}{dt}(\delta q) \quad \text{---- (6)}$$

The second integral of (5) can be solved by integration by parts

$$\int_{t_1}^{t_2} \frac{\partial L}{\partial \dot{q}} \delta \dot{q} \, dt = \int_{t_1}^{t_2} \frac{\partial L}{\partial \dot{q}} \frac{d}{dt}(\delta q) \, dt$$

$$= \frac{\partial L}{\partial \dot{q}} \delta q \Big]_{t_1}^{t_2} - \int_{t_1}^{t_2} \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) \delta q \, dt$$

$$= - \int_{t_1}^{t_2} \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) \delta q \, dt \quad \text{---} (7)$$

because, $\frac{\partial L}{\partial \dot{q}} \delta q \Big]_{t_1}^{t_2} = 0$

Substituting (7) in (5) we get

$$\delta S = \int_{t_1}^{t_2} \left[\frac{\partial L}{\partial q} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right)\right] \delta q \, dt = 0 \quad \text{----} (8)$$

This will be zero only ~~then~~ when the integrand will be zero. Thus we have

$$\frac{\partial L}{\partial q} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) = 0 \quad \text{---} (9)$$

If the system have S degree of freedom then we have the equations of the form

$$\frac{\partial L}{\partial q_i} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) = 0 \quad (i = 1, 2, 3 \text{----} S)$$

This is the second order differential

Q → Discuss addition of velocities under Lorentz transformations.

The transformation equations lead to a relativistic formula for the addition of velocities different from that of classical mechanics.

Let us consider two systems S and S′, the latter moving with uniform relative velocity v along positive x-axis. The velocity of a body is then expressed as follows.

Let the body move through a distance dx in time dt in the system S and through distance dx' in time dt' in the system S′, then

$$\frac{dx}{dt} = u \quad \text{and} \quad \frac{dx'}{dt'} = u' \qquad \text{——— (1)}$$

From Lorentz transformation equations

$$x = \gamma\,(x' + vt') \qquad \text{——— (2)}$$

and

$$t = \gamma\left(t' + \frac{vx'}{c^2}\right) \qquad \text{——— (3)}$$

Differentiating these equations, we get

$$dx = \gamma\,(dx' + v\,dt') \qquad \text{——— (4)}$$

and

$$dt = \gamma\left(dt' + \frac{v \cdot dx'}{c^2}\right) \qquad \text{——— (5)}$$

Dividing (4) by (5) we get

$$\frac{dx}{dt} = \frac{\gamma\,(dx' + v\,dt')}{\gamma\,(dt' + \frac{v \cdot dx'}{c^2})}$$

or
$$\frac{dx}{dt} = \frac{\frac{dx'}{dt'} + v}{1 + \frac{v}{c^2} \cdot \frac{dx'}{dt'}}$$

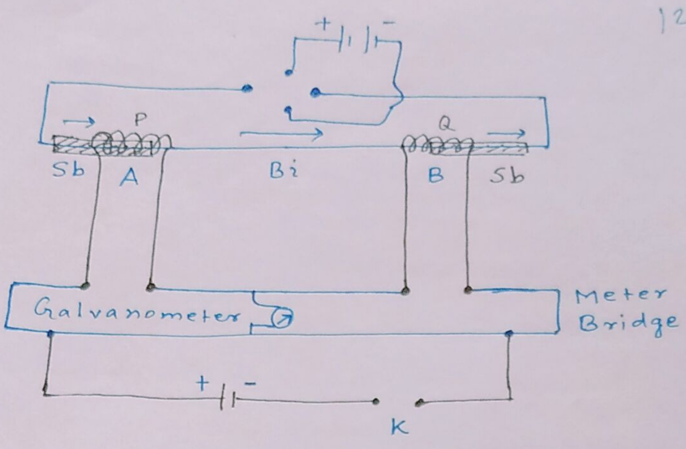
or
$$u = \frac{u' + v}{1 + \frac{v x'}{c^2}} \qquad \text{———— (6)}$$

This is the relativistic law of addition of velocities whereas in classical mechanics it is simply $u = u' + v$, since v is the relative velocity of the two systems. If u' or v is very small as compared with c, then $u = u' + v$ from (6).

The relativistic law of addition of velocities leads to a very important conclusion about the velocity of light c. In the above relation in order that u may be real, u' and v must be both less than c. If we put $u' = c$, then from (6)

$$u = \frac{c + v}{1 + \frac{vc}{c^2}} = \frac{c + v}{1 + \frac{v}{c^2}} = \frac{c + v}{\frac{c + v}{c}} = c$$

method.



(fig. 3.)

In the method, a rod of Bismuth is soldered between two thick antimony rods. The coils P and Q are connected to the two gaps of a meter bridge and the balance point is obtained on the meter bridge wire. Current is passed through the rods from Sb to Bi, the junction A is heated and B is cooled. The Cu-coils P and Q are at different temperatures. Since the electrical resistance of copper wire varies rapidly with change in temperature, the balance point is disturbed and shifts to one side of the meter bridge wire. On reversing the current through the rods, it is found that heat is evolved at the junction B and absorbed at the junction A and the balance point shifts to the other side.

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